Dynamic Competition with Bargaining: Implications for Subsidy and Competition Policies

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Abstract

Existing models of dynamic competition assume that buyers are price-takers, even though prices are negotiated in many industries motivating this literature. We extend a well-known duopoly model to allow for Nash-in-Nash bargaining, with seller price-setting nested as a special case. We illustrate how market structure, dynamic incentives, welfare and the existence of multiple equilibria change with the allocation of bargaining power. We extend the model to allow for forward-looking buyers and more sellers, and to illustrate how the allocation affects optimal subsidy policies and the effects of horizontal mergers, exclusive contracts and policies designed to limit dominance.

Keywords: dynamic competition, learning-by-doing, bargaining power, buyer power, multiple equilibria, dynamic incentives, subsidies, mergers, exclusive contracts.

JEL codes: C73, D21, D43, L13, L41.

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1 Introduction

Industries where features of demand or costs may allow firms to gain future advantages when they make sales are often the focus of government policies. Some policies may aim to speed up the accumulation of advantages, such as industrial and trade policies in sectors like electric vehicle batteries, semiconductors, solar panels and aircraft where learning-by-doing (LBD) is important, while other policies may seek to limit leaders' dominance, such as antitrust interventions targeting mergers, exclusive contracts or certain types of pricing behavior in markets with LBD or network effects.

The models of dynamic competition that have been developed to understand market efficiency and to predict the effects of different policies, either in industrial organization (for example, Fudenberg and Tirole (1983), Cabral and Riordan (1994), Benkard (2000), Besanko, Doraszelski, Kryukov, and Satterthwaite (2010), Besanko, Doraszelski, and Kryukov (2014)) or international trade (Dasgupta and Stiglitz (1988), Leahy and Neary (1999), Neary and Leahy (2000)), assume that sellers unilaterally set prices or quantities. However, buyers are widely reported as negotiating transaction prices in several industries that motivate this literature, and it is unclear how allowing buyers to have some bargaining power might change the literature's conclusions.[1]

We study the effects of negotiation by extending the dynamic duopoly seller model of Besanko, Doraszelski, Kryukov, and Satterthwaite (2010) (BDKS hereafter). The BDKS model assumes that the sellers set prices to compete each period for a myopic buyer, and that sellers' future costs tend to fall when they make a sale as know-how accumulates (LBD), but that know-how may also depreciate stochastically ("forgetting"). Market structure is therefore endogenous, and price competition is dynamic. Assuming thirty levels of know-how ($M = 30$) for each firm, BDKS use numerical homotopies to identify as many Markov Perfect Nash equilibria (MPNEs) as possible for different parameters. They show that dynamics can lead to the existence of multiple equilibria, which may support qualitatively different market

[1] For example, press reports suggest the ability of buyers, especially larger ones, to negotiate lower prices for electric vehicle batteries (`https://about.bnef.com/blog/behind-scenes-take-lithium-ion-battery-prices/`), commercial aircraft (`https://simpleflying.com/how-much-do-boeing-aircraft-cost/`) and semiconductors (`https://www.linkedin.com/pulse/latest-news-semiconductor-foundries-negotiating-price-quincy-qiu/`).

structures, for many parameters.

The first part of our paper analyzes what happens when we replace BDKS's price-setting game with a form of generalized Nash-in-Nash bargaining over prices (Horn and Wolinsky (1988), Collard-Wexler, Gowrisankaran, and Lee (2019)). Our model nests seller price-setting when the buyer bargaining weight parameter, τ, is equal to zero.

This part of the paper makes several novel contributions. First, we provide a reformulation of the equations characterizing symmetric MPNEs which reduces the number of equations and unknowns significantly, from $2M^2$ to $\frac{M(M-1)}{2}$. This allows us to derive analytical comparative statics for how outcomes vary with τ in a special case of the model if $M = 2$, and to use a non-homotopy method to find all equilibria when $M = 3$. This allows us to provide a novel confirmation of homotopy results, and the reformulation also provides a convenient way to analyze subsidy policies. Second, we show that, when $M = 3$ or $M = 30$, multiplicity is eliminated, for all cost technologies, when buyers have more than moderate bargaining power ($\tau \geq 0.25$). Third, we analyze how the comparative statics from the $M = 2$ case generalize. We find that, when LBD effects are significant and deprecation is not too likely, expected market concentration typically increases in τ and that discounted total surplus (efficiency) typically increases and then decreases. Fourth, we identify how bargaining power and dynamics interact in ways that we have not seen discussed elsewhere, and which are not captured by intuitions for how bargaining affects outcomes in static models. For example, while sellers claim a smaller share of surplus in a given negotiation when τ increases, their dynamic incentives can substantially increase with τ when τ is small, as this same effect makes leaders more likely to make future sales so that the value of becoming a leader rises.

The second part of the paper extends the model, focusing on example parameters where LBD is quite important and depreciation is not too likely, leading to several additional findings. First, we show that the qualitative patterns in the first part of the paper are robust to allowing for up to four sellers, and to allowing for forward-looking repeat buyers, using an approach developed in Sweeting, Jia, Hui, and Yao (2022) (SJHY). Second, we find new results in several policy applications that should be of independent interest. For example, we show that the structure of efficiency-maximizing subsidies can be very sensitive to τ, and that policies that would maximize efficiency when $\tau = 0$ may reduce efficiency for values of τ as low as 0.05. Our other examples consider several stylized competition policy applications. For instance, we illustrate how the profitability, and the sign of the welfare effects, of a horizontal merger, and the effects of policies to limit dominance, can depend on τ. We also use the model to consider whether an exclusive contract between a seller and a large repeat customer causes anticompetitive foreclosure effects in the market for non-contracted customers.

After a discussion of the related literature, the rest of the paper is structured as follows. Section 2 explains how we extend the BDKS model to allow for bargaining. Section 3 details our outcome measures and the two alternative formulations of the equilibrium equations. Section 4 shows how the bargaining weight parameter affects equilibrium outcomes when we maintain all of BDKS's other assumptions. The sub-sections of Section 5 present our extensions and policy applications. Section 6 concludes. The online Appendices contain proofs, computational details, and supporting results.

Related Literature. Our paper contributes directly to the theoretical literature studying dynamic price competition with LBD, including the analytical analysis of Cabral and Riordan (1994) and the computational results of BDKS, Besanko, Doraszelski, and Kryukov (2014), Besanko, Doraszelski, and Kryukov (2019a), Besanko, Doraszelski, and Kryukov (2019b) and SJHY, by examining how bargaining over prices affects many of the outcomes considered in this literature.[2,3] Benkard (2004), Kim (2014), Kalouptsidi (2018), An and Zhao (2019), and Barwick, Kalouptsidi, and Zahur (2019) use estimated models of particular industries to consider the effects of mergers, trade and subsidy policies. Our policy examples are based on more stylized models but they suggest that some conclusions in the empirical literature may be sensitive to how prices are assumed to be set.

Another literature has introduced Nash-in-Nash bargaining into static models of business-

[2]The Besanko, Doraszelski and Kryukov papers, and SJHY, use a model (the "BDK model") where sellers can enter or exit, and there is no forgetting. Reallocating bargaining power away from sellers in the BDK model might cause the industry to disappear entirely, which is a separate effect from the effects on dynamic competition which we want to focus on here.

[3]A distinct literature on industry dynamics (Abbring and Campbell (2010) and Abbring, Campbell, Tilly, and Yang (2018)) assumes that firms are symmetric to guarantee equilibrium uniqueness in order to simplify the analysis of how industries respond to external aggregate shocks.

to-business transactions in healthcare (for example, Gowrisankaran, Nevo, and Town (2015), Ho and Lee (2017), Grennan (2013)), cable television (Crawford, Lee, Whinston, and Yurukoglu (2018))) and consumer packaged goods settings (Draganska, Klapper, and Villas-Boas (2010)). The interactions between bargaining power and dynamics that we emphasize cannot occur in this literature.

A smaller set of papers has considered bargaining in dynamic settings. Lee and Fong (2013) assume period-by-period Nash-in-Nash bargaining in a game where hospitals and insurance companies form networks that change stochastically, while Dorn (2024) shows how Kalai (1977) proportional bargaining facilitates consideration of multiperiod hospital-insurer contracts with index clauses. Yang (2020) embeds Nash-in-Nash bargaining over prices into a dynamic model of innovation in the smartphone device industry and Tiew (2024) considers a dynamic game where duopoly newspapers may bargain over forming or continuing a joint operating agreement.[4] However, in our paper, the allocation of bargaining power affects sales, which, through LBD, leads to effects on future competition and dynamic incentives, which feedback into equilibrium prices. This introduces elements of dynamic competition that do not appear in these papers.

2 Model

The model is an infinite horizon, discrete time, discrete state game. The common discount factor is $\beta = \frac{1}{1.05}$.[5]

Sellers, States and Technology. There are two long-lived ex-ante symmetric but differentiated sellers ($i = 1, 2$). i's production cost is $c(e_i) = \kappa\rho^{\log_2(\min(e_i,m))}$, where $e_i = 1, ..., M$ is a commonly observed state variable indicating i's "know-how".[6] $\rho \in (0, 1)$ and lower values

[4] We also contribute to the small literature that uses computational dynamic models to consider policy questions. In addition to several of the papers cited above, Gowrisankaran (1999) and Mermelstein, Nocke, Satterthwaite, and Whinston (2020) consider some of the trade-offs involved in merger policy.

[5] As BDKS discuss, one can interpret this discount factor as corresponding to a period length of 1 month, a monthly discount rate of 1 percent and a probability that the industry survives each month of 0.96.

[6] Asker, Fershtman, Jeon, and Pakes (2020), Sweeting, Roberts, and Gedge (2020) and Sweeting, Tao, and Yao (2024) consider dynamic models where state variables are private information.

imply stronger learning economies.[7] The industry state is $\mathbf{e} = (e_1, e_2)$. A firm is called a "leader" when it has strictly higher know-how than its "laggard" rival. BDKS assume $m = 15$ and $M = 30$, but we will also consider some smaller state spaces.

As described below, one of the sellers will sell one unit each period. e_i evolves, except at the state space boundaries, according to

$$e_{i,t+1} = e_{i,t} + q_{i,t} - f_{i,t} \tag{1}$$

where $q_{i,t}$ is equal to one (zero) if and only if firm i (j) makes the sale, and $f_{i,t}$ is equal to one (0 otherwise) with probability $\Delta(e_i) = 1 - (1-\delta)^{e_i}$ with $\delta \in [0, 1)$.[8] Δ increases in both δ and e_i. A firm's know-how can go down only when it fails to make a sale.

Buyers. Every period a buyer arrives who will purchase exactly one unit from one of the sellers. The buyer is in the market only once, and she chooses the seller that maximizes her indirect utility, $v - p_i + \sigma\varepsilon_i$, where p_i is the price paid to the chosen seller, and the ε_is are i.i.d. private information Type I extreme value payoff shocks. σ parameterizes seller product differentiation.

Bargaining. BDKS assume sellers make simultaneous take-it-or-leave-it price offers. Instead, we assume a form of "Nash-in-Nash" bargaining. Suppose that, before the εs are realized, the buyer sends separate agents to each seller, and that each agent-seller pair negotiates, in a Nash bargain where the buyer bargaining weight is τ (seller weight $1 - \tau$), a price at which the buyer can purchase from that seller, taking the price negotiated in the other negotiation as given.[9] Once negotiations are complete, the buyer observes the εs and chooses which seller to buy from, with a purchase possible only if a price was agreed. Choice probabilities are, therefore, given by the standard logit formula given the agreed prices. If $\tau = 1$, the buyer has "all of the bargaining power" and extracts all of the expected surplus

[7] The assumption that $\rho < 1$ eliminates the possibility that firms would have the same costs in different know-how states.

[8] When $e_{i,t} = 1$ and $q_{i,t} = 0$, $e_{i,t+1} = 1$, and when $e_{i,t} = M$ and $q_{i,t} = 1$, $e_{i,t+1} = M$.

[9] The distinction from many other applications is that the agreement of a price does not mean that a unit will necessarily be traded. In our model, agreement provides the buyer with an option to buy.

from an agreement. BDKS's price-setting assumption is replicated if $\tau = 0$ (sellers have all of the bargaining power).

Parameters. In the text, we will assume $\sigma = 1$, $\kappa = 10$ and $v = 10$, although, with no outside good, the value of v does not affect equilibrium strategies and, as explained below, it does not enter our surplus calculations. Our focus will therefore be on τ, and the technology parameters ρ and δ.

If $\tau \neq 0$, what values of τ are "relevant"? $\tau = 0.5$ is often used in applications. In static models, outcomes when $\tau = 0.5$ are often close to the averages of outcomes when $\tau = 0$ and $\tau = 1$. We will show that this is often not true in our dynamic model.

On the other hand, if one interprets myopic buyers as also being short-lived and impatient, then one might view τs close to zero as being the only ones that are economically rationalizable. Our finding that outcomes can change quite sharply even when τ moves from zero to small values like 0.1 would then be of more interest. However, we do not interpret the myopia assumption as necessarily implying that buyers are shorter lived or less patient than the sellers. For example, a large corporation that only buys a complex and durable machine very infrequently may ignore how its purchase affects future competition between machine producers (the key implication of myopia in our paper), but it may be willing to negotiate patiently for a lower price.

3 Equilibrium and Outcomes

The equilibrium concept is symmetric and stationary Markov Perfect Nash equilibrium (MPNE, Maskin and Tirole (2001), Ericson and Pakes (1995), Pakes and McGuire (1994)). This section describes two alternative characterizations of equilibria and our measures of outcomes and incentives.

3.1 Standard Formulation of Equilibrium Conditions.

BDKS specify equations for equilibrium seller values, $V^{S*}(\mathbf{e})$, defined at the start of each period, and equilibrium prices, $p^*(\mathbf{e})$. We extend their formulation to allow for bargaining.

Symmetry implies that we only need to define equations for the prices and values of seller 1, i.e., $p_2^*(e_1,e_2) = p_1^*(e_2,e_1)$ and $V_2^{S*}(e_1,e_2) = V_1^{S*}(e_2,e_1)$.

Beginning of period value for firm 1 (V^S):

$$V_1^{S*}(\mathbf{e}) - D_1^*(\mathbf{e})(p_1^*(\mathbf{e}) - c(e_1)) - \sum_{k=1,2} D_k^*(\mathbf{e})\mu_{1,k}^S(\mathbf{e}) = 0, \tag{2}$$

where $\mu_{1,k}^S(\mathbf{e})$ is seller 1's continuation value when seller k makes the sale,

$$\mu_{1,k}^S(\mathbf{e}) = \beta \sum_{\forall e'_{1,t+1}|e_{1,t}} \sum_{\forall e'_{2,t+1}|e_{2,t}} V_1^{S*}(e'_{1,t+1}, e'_{2,t+1}) \Pr(e'_{1,t+1}|e_{1,t},k) \Pr(e'_{2,t+1}|e_{2,t},k), \tag{3}$$

and $\Pr(e'_{i,t+1}|e_{i,t},k)$ is the probability that i's state transitions from $e_{i,t}$ to $e'_{i,t+1}$ when $q_{k,t} = 1$. Given prices, the probability that seller k makes the sale is

$$D_k(p_1,p_2) = \frac{\exp\left(\frac{v-p_k}{\sigma}\right)}{\exp\left(\frac{v-p_1}{\sigma}\right) + \exp\left(\frac{v-p_2}{\sigma}\right)}. \tag{4}$$

$D_k^*(\mathbf{e})$ is the choice probability given equilibrium prices, $p_1^*(\mathbf{e})$ and $p_2^*(\mathbf{e})$.

Negotiated prices (p): Our Nash-in-Nash bargaining assumption implies that, given a p_2, $p_1(p_2,\mathbf{e})$ will be determined as

$$\begin{aligned} p_1(p_2,\mathbf{e}) = \arg\max_{p_1} \left[CS(p_1,p_2) - CS(p_2)\right]^{\tau} \times ... \\ \left[D_1(p_1,p_2)(\mu_{1,1}^S(\mathbf{e}) + p_1 - c(e_1)) + (1 - D_1(p_1,p_2))\mu_{1,2}^S(\mathbf{e}) - \mu_{1,2}^S(\mathbf{e})\right]^{(1-\tau)} \end{aligned} \tag{5}$$

where $CS(p_1,p_2) = \sigma \log\left(\sum_{k=1,2} \exp\left(\frac{v-p_k}{\sigma}\right)\right)$ (i.e., the expected surplus of the buyer when it is able to choose from both firms). $CS(p_2) = v - p_2$ is the buyer's expected surplus if there is no agreement with seller 1 and seller 2 is the buyer's only option. Given an equilibrium p_2^* and continuation values, equilibrium $p_1^*(\mathbf{e})$ will therefore solve the first-order condition

$$\begin{aligned} \tau \frac{\partial CS(p_1^*(\mathbf{e}), p_2^*)}{\partial p_1} \left[D_1^*(\mathbf{e})(\mu_{1,1}^S(\mathbf{e}) + p_1^*(\mathbf{e}) - c(e_1)) + (1 - D_1^*(\mathbf{e}))\mu_{1,2}^S(\mathbf{e}) - \mu_{1,2}^S(\mathbf{e})\right] + ... \\ (1-\tau)\left[CS(p_1^*(\mathbf{e}),p_2^*) - CS(p_2^*)\right]\left[D_1^*(\mathbf{e}) + \frac{\partial D_1^*(\mathbf{e})}{\partial p_1}\left(p_1^*(\mathbf{e}) - c(e_1) + \mu_{1,1}^S(\mathbf{e}) - \mu_{1,2}^S(\mathbf{e})\right)\right] = 0, \end{aligned} \tag{6}$$

where $\frac{\partial CS(p_1^*(\mathbf{e}),p_2^*)}{\partial p_1} = -D_1^*(\mathbf{e})$ and $\frac{\partial D_1^*(\mathbf{e})}{\partial p_1} = -\frac{D_1^*(\mathbf{e})(1-D_1^*(\mathbf{e}))}{\sigma}$. Algebraic manipulation shows that this can be simplified to

$$-\tau D_1^*(\mathbf{e})\left[p_1^*(\mathbf{e}) - \widehat{c}_1(\mathbf{e})\right] + (1-\tau)\left[\sigma - (1 - D_1^*(\mathbf{e}))(p_1^*(\mathbf{e}) - \widehat{c}_1(\mathbf{e}))\right] \log \frac{1}{1 - D_1^*(\mathbf{e})} = 0. \quad (7)$$

$\widehat{c}_1(\mathbf{e}) = c(e_1) - (\mu_{1,1}^S(\mathbf{e}) - \mu_{1,2}^S(\mathbf{e}))$ is seller 1's economic opportunity cost of a sale, which we will discuss in more detail below. The first-order condition is identical to BDKS if $\tau = 0$.

3.2 Formulation of Equilibrium Conditions in Terms of Buyer Choice Probabilities.

The equilibrium choice probability equations can be written as

$$\sigma \log\left(\frac{1}{D_1^*(\mathbf{e})} - 1\right) - p_1^*(\mathbf{e}) + p_2^*(\mathbf{e}) = 0. \quad (8)$$

We now show that prices can be expressed as functions of D_1^*s and parameters only.

From equation (7), firm 1's markup over its opportunity cost is

$$p_1^*(\mathbf{e}) - \widehat{c}_1(\mathbf{e}) = \Phi(D_1^*(\mathbf{e})) = \frac{(1-\tau)\sigma \log \frac{1}{1-D_1^*(\mathbf{e})}}{\tau D_1^*(\mathbf{e}) + (1-\tau)(1 - D_1^*(\mathbf{e})) \log \frac{1}{1-D_1^*(\mathbf{e})}}. \quad (9)$$

Corollary 1 of Proposition 1 will show that, given $\widehat{c}$s, these equilibrium markups are unique and monotonic in τ.

Expressing the seller 1 state transition matrix when k makes a sale as $\mathbf{Q_k}$ and the stacked markup vector as $\mathbf{\Phi}(\mathbf{D_1})$,

$$\widehat{\mathbf{c}}_\mathbf{1} = \mathbf{c_1} - \beta(\mathbf{Q_1} - \mathbf{Q_2})\mathbf{V_1^S}, \quad (10)$$

and

$$\mathbf{V_1^S} = (\mathbf{I} - \beta\mathbf{Q_2})^{-1}\left[\mathbf{D_1} \circ \mathbf{\Phi}(\mathbf{D_1})\right]. \quad (11)$$

where $\circ$ denotes the element-wise product of two vectors. Therefore,

$$\mathbf{p_1} = \mathbf{\Phi}(\mathbf{D_1}) + \mathbf{c_1} - \beta(\mathbf{Q_1} - \mathbf{Q_2})(\mathbf{I} - \beta\mathbf{Q_2})^{-1}[\mathbf{D_1} \circ \mathbf{\Phi}(\mathbf{D_1})]. \quad (12)$$

which can be substituted into (8), giving equations in D_1^* only.

The assumptions of symmetry and no outside good imply $D_1^*(e,e) = \frac{1}{2} \forall e$ and $D_1^*(e,e') = 1 - D_1^*(e',e)$, so the problem reduces to $\frac{M(M-1)}{2}$ equations and unknowns. This compares to $2M^2$ equations for prices and seller values in the standard formulation. For $M = 30$, our new formulation reduces the problem from 1,800 equations to 435, and for $M = 3$ the reduction is from 18 equations to only 3. For $M = 2$, the reduction is from 8 to 1.

3.3 Social Planner Problem.

As an efficiency benchmark, we will compute the choice probabilities of a social planner buyer who, each period, chooses which seller to buy from to maximize, using the β discount factor, the discounted sum of current and future buyer utility less production costs.

$$D_1^{SP}(\mathbf{e}) = \frac{1}{1 + \exp\left(\frac{c_1(\mathbf{e}) - c_2(\mathbf{e}) + \mu_2^{SP}(\mathbf{e}) - \mu_1^{SP}(\mathbf{e})}{\sigma}\right)} \tag{13}$$

where μ_i^{SP} is the discounted continuation surplus when seller i is chosen. In vector form,

$$\mu_{\mathbf{2}}^{\mathbf{SP}} - \mu_{\mathbf{1}}^{\mathbf{SP}} = \beta(\mathbf{Q_2} - \mathbf{Q_1}) \left(\mathbf{I} - \beta \sum_{k=1,2} \mathbf{D_k^{SP}} \circ \mathbf{Q_k}\right)^{-1} \sum_{k=1,2} \left[\mathbf{D_k^{SP}} \circ \left(\sigma \log \frac{1}{\mathbf{D_k^{SP}}} + v - \mathbf{c_k}\right)\right], \tag{14}$$

Substituting (14) into (13) provides $\frac{M(M-1)}{2}$ equations for the asymmetric state choice probabilities, the D_1^{SP}s.

3.4 Static Equilibrium Outcomes.

We will make two alternative types of comparisons to outcomes that might be viewed as static. One comparison is to a model where cost transitions are the same as in the full model but both sellers do not consider dynamic incentives when bargaining with buyers. For clarity, we will refer to “static sellers” when making this comparison. The second comparison is to a model where costs are also assumed to be fixed over time so there is no LBD and no depreciation, so that even a forward-looking seller would use a static strategy. We will refer to this case as “fixed production costs”.

3.5 Numerical Methods.

Most of our analysis will require us to solve for equilibria, or planner outcomes, numerically. A single equilibrium can be found by solving either formulation of the equilibrium equations, using the Pakes and McGuire (1994) algorithm or, if $\delta = 0$, using backwards induction. To track what happens to equilibrium outcomes when we vary τ, holding the technology parameters fixed, we use BDKS's numerical homotopy method to trace the equilibrium correspondence (a "τ-homotopy"). When we want to try to identify all equilibria, we also follow BDKS by criss-crossing the parameter space using homotopies, starting new homotopies when additional equilibria are found.[10]

Homotopies are not guaranteed to identify all equilibria in this model, and, in practice, they often "stall" and need to be restarted from a nearby equilibrium. Therefore, we acknowledge that, when we use homotopies, we may have missed equilibria for any particular set of parameters. However, we are confident in the broad sweep of our results. Appendix B.2 describes a novel alternative algorithm for finding all equilibria in the $M = 3$ model, and that method finds identical sets of equilibria as the homotopy method for all technology parameters.[11]

3.6 Outcomes of Interest.

Most of our analysis will focus on the following measures of equilibrium outcomes, assuming, unless otherwise stated, that the game begins in state (1,1) in period $t = 1$.

Concentration. Following BDKS, we measure expected market concentration in period t as

$$HHI^t = \sum_{\forall \mathbf{e}} \lambda^t(\mathbf{e}) HHI(\mathbf{e}) \text{ where } HHI(\mathbf{e}) = \sum_{k=1,2} \left(\frac{D_k^*(\mathbf{e})}{D_1^*(\mathbf{e}) + D_2^*(\mathbf{e})} \right)^2$$

and $\lambda^t(\mathbf{e})$ is the probability that the state will be $\mathbf{e}$ in the t^{th} period. The minimum HHI^t is 0.5. We will focus primarily on what we see as medium- to long-run concentration measures

[10] For a given τ, we use ρ- and δ- homotopies, but we have also searched extensively using τ-homotopies, and most of our figures in this paper are also based on the output of τ-homotopies.

[11] In the context of the BDK model, SJHY develop a third method that can be used to identify whether particular types of equilibrium exist. That method also gives results that are consistent, for all technology parameters, with the equilibria identified by homotopies.

for a given M (e.g. $t = 32$, and $t = 200$ when $M = 30$). However, readers should understand that HHIs may converge very slowly to long-run values. For example, when $m = 15$, $M = 30$, $\rho = 0.75$ and $\delta = 0.023$, $HHI^t \approx 0.5$ for $t > 4,000$, but $HHI^{1,000} \approx 0.6$.

Surplus. The expected total surplus in period t, TS^t, is calculated as the expected ε of the purchased good less the production cost. We exclude v from the calculation as it does not affect strategies, which leads to the reported TS^ts being negative. TS^{PDV} is the present discounted value (PDV) of expected total surplus at the start of the game given equilibrium play. Buyer surplus (CS) is measured as the expected ε less the price paid, and producer surplus (PS) as the sum, across sellers, of expected revenues less expected production costs. To simplify the exposition, we will primarily focus on total surplus.

3.7 Dynamic Incentives.

From equation (9), equilibrium prices equal a static markup term, which depends directly on τ, plus the seller's opportunity cost, $\widehat{c_1}(\mathbf{e}) = c(e_1) - (\mu^S_{1,1}(\mathbf{e}) - \mu^S_{1,2}(\mathbf{e}))$. The terms in parentheses reflect how seller 1's future value increases when it, rather than seller 2, makes the sale. These terms therefore capture how dynamic incentives affect pricing. Bargaining in future states will affect these opportunity costs.

Besanko, Doraszelski, and Kryukov (2014) decompose dynamic incentives as

$$\mu^S_{1,1}(\mathbf{e}) - \mu^S_{1,2}(\mathbf{e}) = \underbrace{\mu^S_{1,1}(\mathbf{e}) - \mu^S_{1,0}(\mathbf{e})}_{\text{“Advantage Building Incentive” } (AB(\mathbf{e}))} + \underbrace{\mu^S_{1,0}(\mathbf{e}) - \mu^S_{1,2}(\mathbf{e})}_{\text{“Advantage Denying Incentive” } (AD(\mathbf{e}))} \tag{15}$$

where $\mu^S_{1,0}(\mathbf{e})$ is seller 1's continuation value if, hypothetically, the buyer were to purchase from *neither* seller. From equation (9), increases in either dynamic incentives will lower prices (all else equal). We will sometimes report the PDV of dynamic incentives given equilibrium play. For example,

$$AB^{PDV} = \sum_{t=1}^{\infty} \beta^t \sum_{\mathbf{e}} \lambda_t(\mathbf{e}) \left[AB(\mathbf{e}) + AB(\mathbf{e}')\right]. \tag{16}$$

where $\mathbf{e}'$ switches the states of the sellers. The PDV of all dynamic incentives adds AB^{PDV} and AD^{PDV} together.

4 Bargaining Weights, Equilibrium Outcomes and Incentives

This section shows how equilibrium outcomes vary with the bargaining weight parameter τ. To emphasize, bargaining is the only change from BDKS's model at this point, and the model is identical to BDKS's if $\tau = 0$. We discuss what can be said about uniqueness and comparative statics when there are no dynamics or dynamic incentives are held fixed, before discussing the existence of multiple equilibria and comparative statics in the dynamic model.

4.1 Equilibria in the Static Model.

We begin by considering the price equilibrium in a one-shot version of the game with fixed production costs. In this case, equilibrium prices will be solutions to the first-order conditions (9), given demand, with $\widehat{c_1} = c_1$ and $\widehat{c_2} = c_2$.

Proposition 1 *Consider a one-shot bargaining game with seller costs c_1 and c_2,*

1. *for any $\tau \in [0, 1]$, equilibrium prices are unique (i.e., unique (p_1^*, p_2^*) will satisfy equations (4) and (7), or equivalently (8) and (9)).*

2. *the equilibrium markups, $p_i^* - c_i$, and therefore prices, of both firms, strictly decrease in τ. If $c_i > c_j$, then the markup of firm j decreases more than the markup of i.*

3. *if $c_i > c_j$, then $D_i^* < \frac{1}{2}$ and D_i^* is strictly decreasing in τ, and D_j^* is strictly increasing in τ. If $c_i = c_j$, then $D_i^* = \frac{1}{2}$ for all τ.*

4. *expected buyer surplus monotonically increases in τ, and expected producer surplus monotonically decreases in τ. If $c_i \neq c_j$ then expected total surplus monotonically increases in τ.*

Proof. See online Appendix A.1. ■

4.2 Statewise Uniqueness of Equilibria in the Dynamic Model.

A straightforward corollary is that parts 1-3 of Proposition 1 will hold for the equilibrium in any state of the dynamic game if continuation values are held fixed. This extends BDKS's

Proposition 9 ("statewise uniqueness"), shown for the price-setting case, to our model with additional comparative statics.

Corollary 1 *Consider a state (e_1, e_2) and hold fixed seller opportunity costs $\widehat{c}_1$ and $\widehat{c}_2$,*

1. *for any $\tau \in [0, 1]$, equilibrium prices are unique (i.e., unique (p_1^*, p_2^*) will satisfy equations (4) and (7), or equivalently (8) and (9)).*

2. *the equilibrium markups, $p_i^* - \widehat{c}_i$, and therefore prices, of both firms, strictly decrease in τ. If $\widehat{c}_i > \widehat{c}_j$, then the markup of firm j decreases more than the markup of i.*

3. *if $\widehat{c}_i > \widehat{c}_j$, then $D_i^* < \frac{1}{2}$ and D_i^* is strictly decreasing in τ, and D_j^* is strictly increasing in τ. If $\widehat{c}_i = \widehat{c}_j$, then $D_i^* = \frac{1}{2}$ for all τ.*

4. *expected buyer surplus is monotonically increasing in τ.*

Proof. This corollary follows immediately from parts 1-3 of Proposition 1, replacing $\widehat{c}_1$ and $\widehat{c}_2$ with c_1 and c_2, and the buyer surplus result from part 4. ■

4.3 Existence of Multiple Equilibria.

Statewise uniqueness does not guarantee equilibrium uniqueness in the full model as different equilibria would cause continuation values to differ. Extending BDKS's Proposition 3, equilibrium uniqueness can be shown only in limited special cases.

Proposition 2 *In a model with any $m \leq M$, there will be a unique symmetric MPNE if*

1. *$\delta = 0$, for any ρ and τ, or*

2. *$\tau = 1$, for any ρ and δ.*

Proof. See online Appendix A.2. ■

For the $m = M = 2$ model that we consider below, the equilibrium can be characterized by a single equation, and we never find multiplicity for any ρ or δ parameters. BDKS use homotopies to identify multiple equilibria for many (ρ, δ) combinations when $0 < \delta < 1$, $\tau = 0$, $M = 30$ and $m = 15$. We repeat their analysis for values of τ from 0 to 1 in increments of 0.05 for the ($m = 15$, $M = 30$) model, which we will call the "$M = 30$" model in the following, and an $m = M = 3$ ("$M = 3$") model.

For the $M = 3$ model, we conduct a thorough search for the full range of technologies, although, in this subsection, our diagrams focus on $0 \leq \delta \leq 0.2$, as we never find multiplicity for $\delta > 0.2$. We find an identical set of equilibria using our alternative method (Appendix B.2). For the $M = 30$ model, we report results and have searched most extensively for parameters $0.6 \leq \rho \leq 1$, covering all of the estimated progress ratios from the 97 empirical studies considered by Ghemawat (1985), and $0 \leq \delta \leq 0.2$, where the upper bound implies almost certain know-how depreciation for $e_i \geq 20$.[12] For $\tau = 0$, we find almost identical results to BDKS, except that we identify some additional multiplicity for a very small range of parameters where ρ is very close to 1.

Figure 1(a) and (c) show, for a fine grid of (ρ, δ)s, the smallest τs for which we identify multiplicity (white if we never do). For $M = 30$, we follow BDKS in using a log scale for δ, so more focus is given to technologies that imply less depreciation. While there are technologies for which multiplicity exists when $\tau = 0.05$, 0.1 or 0.15, but not when $\tau = 0$, these cases are limited and they are adjacent to technologies where price-setting also supports multiplicity.

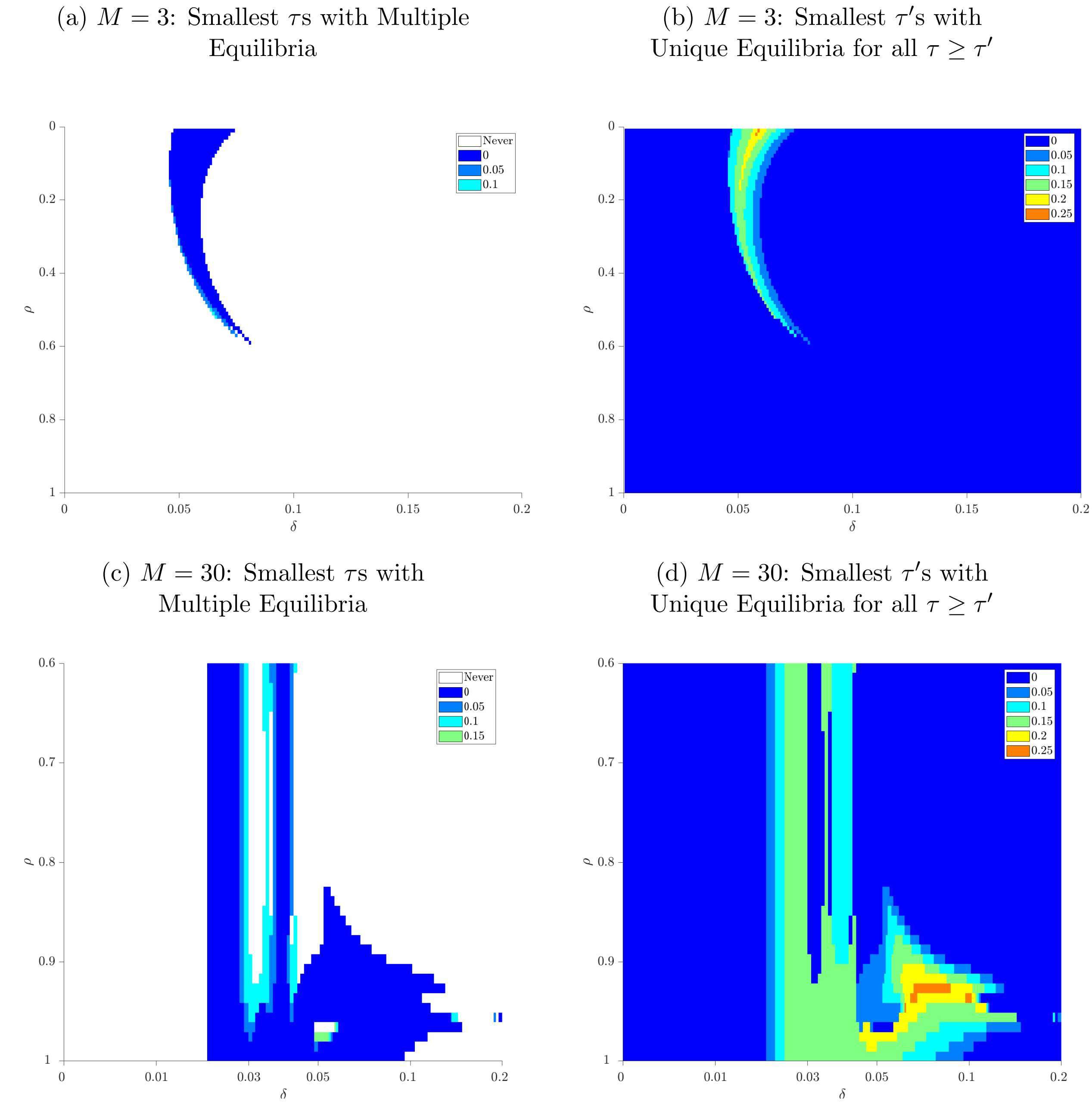
Panels (b) and (d) show the smallest values of τ for which equilibria are unique for all weakly larger τs. For both the $M = 3$ and $M = 30$ models, there is no multiplicity for any technologies when $\tau \geq 0.25$.

In much of the applied literature, multiplicity is viewed as a problem that complicates estimation and the analysis of counterfactuals.[13] From this perspective, the fact that multiplicity is eliminated when assuming a price formation process that may be more realistic for some industries is encouraging for future research.

[12] We have also not identified multiplicity for $\tau \geq 0.25$ outside of this region, although our search has not been so thorough.

[13] An alternative viewpoint, which we also share, is that multiplicity reflects that demand and cost primitives can be insufficient to determine outcomes, so that additional forces that economists have identified, such as expectations (Cass and Shell (1983)), can play a significant role.

Figure 1: Equilibrium Multiplicity in $M = 3$ and $M = 30$ Models Based on Homotopy Analysis for τ in 0.05 Increments. For the $M = 3$ models we only report the results for $0 \leq \delta \leq 0.2$, as we identify no multiplicity for higher δs.



4.4 Bargaining Weights and Equilibrium Outcome Comparative Statics.

We now address how equilibrium prices and market outcomes change with the buyer bargaining weight parameter τ. We discuss analytical comparative statics when $m = M = 2$ and there is no forgetting, and we then use an $M = 3$ example to illustrate how τ affects outcomes and incentives when forgetting is possible. We then summarize whether the identified comparative statics hold across other technologies for both $M = 3$ and $M = 30$. We find that they typically do as long as depreciation is not too likely.

$\mathbf{m = M = 2}$ Analytical Comparative Statics. When $M = 2$, our alternative formulation reduces to a single equation in $D_1^*(1,2)$. If we assume $\delta = 0$, then we can prove the following results.

Proposition 3 *If $m = M = 2$ and $\delta = 0$, the unique symmetric equilibrium will have the following properties*

1. $D_1^*(1,2)(\tau) < \frac{1}{2}$ *for all* τ.
2. $D_1^*(1,2)(\tau)$ *is strictly decreasing in* τ.
3. *for* $t \geq 2$, HHI^t *is strictly increasing in* τ.
4. *there exists a* τ^* *such that* $TS^{PDV}(\tau^*) = TS^{SP}$, $TS^{PDV}(\tau^*)$ *is strictly increasing in* τ *for* $\tau \in (0, \tau^*)$ *and strictly decreasing in* τ *for* $\tau \in (\tau^*, 1)$.

Proof. See online Appendix A.3. ■

As τ increases, the laggard is less likely to make a sale, i.e., the comparative static in parts 3 of Proposition 1 and Corollary 1 holds even though opportunity costs are endogenous. As a result, the laggard becomes less likely to catch up and the asymmetric market structure becomes more likely.

Myopic buyers maximize their current period surplus. In a static model (i.e., a one-shot game with fixed production costs), total surplus will therefore be maximized, and what Besanko, Doraszelski, and Kryukov (2019a) term the "PR distortion" will be eliminated, when $\tau = 1$ and prices equal production costs.[14] Further, in this case, Proposition 1 shows that total surplus monotonically increases in τ. However, in a dynamic model, efficiency also depends on how market structure evolves, with the loss of welfare because of a failure to evolve optimally creating the "MS distortion". Part 4 of Proposition 3 reflects how, when $\tau = 1$, one can show that a myopic buyer will be too likely to buy from the leader in state (1,2), i.e., $D_1^*(1,2)$ will be too low, because it ignores how future buyers may benefit from a second low cost option, whereas, when $\tau = 0$, the buyer will be too likely to buy from the laggard, i.e., $D_1^*(1,2)$ will be too high, because of the leader's larger markup.

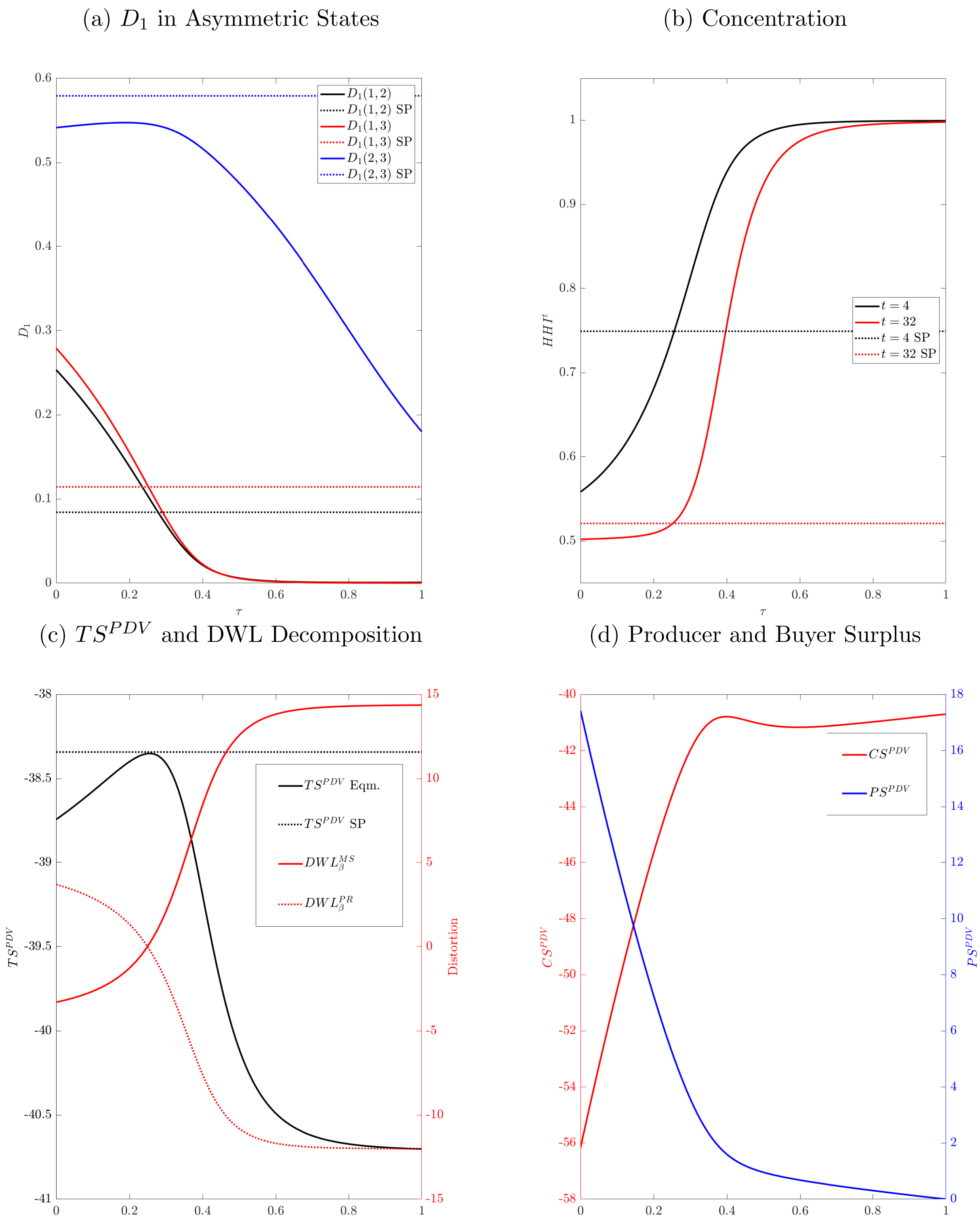
The assumption that depreciation cannot happen is obviously limiting, but the same comparative statics can be shown to hold for any δ in a slightly adjusted model where there is no depreciation when $e_i = 1$ (i.e., if an $e_i = 1$ firm makes a sale it will advance to know-how level 2 for sure). We will see below that the comparative statics also hold in our game with more states for many parameters where the probability of depreciation in low know-how states is small enough.

An $\mathbf{m = M = 3}$ Illustrative Example with $\rho = 0.3$ and $\delta = 0.03$. We will use these "$M = 3$ example" parameters extensively below. $\rho = 0.3$ implies that $c(1) = 10$, $c(2) = 3$ and $c(3) = 1.483$, so there is significant LBD. $\delta = 0.03$ implies that $\Delta(1) = 0.03$, $\Delta(2) = 0.059$ and $\Delta(3) = 0.087$, so that firms are never too likely to lose their know-how when they fail to make a sale. These parameters support a unique equilibrium for all τ, simplifying our exposition. Appendix C.1 provides some additional information on prices, choice probabilities and market structure for the social planner outcome and the polar cases where $\tau = 0$ and $\tau = 1$.

Choice Probabilities, Concentration and Welfare Comparative Statics. Figures 2(a) shows the equilibrium probabilities of the laggard making the sale in asymmetric states as a function τ, with the social planner's probabilities shown for comparison. The laggard's sale probabilities in states (1,2) and (1,3) exceed the social planner's probabilities when $\tau = 0$ and, similar to the prediction of part 2 of Proposition 3, they decline monotonically in τ. However, different to Proposition 3, $D_1^*(2,3)$ is both greater than $\frac{1}{2}$ and too low when $\tau = 0$, and it initially

[14] See footnote 16 for a formal definition of these distortions.

Figure 2: Equilibrium Sale Probabilities, Concentration and Welfare as a Function of τ for $M = 3$ model with $\rho = 0.3$ and $\delta = 0.03$. SP = social planner outcome, DWL = deadweight loss. Homotopies and our alternative method establish that there is a unique equilibrium for every τ.



increases in τ, before falling monotonically if $\tau > 0.35$.

Similar to part 3 of Proposition 3, concentration, measured by either HHI^4 or HHI^{32}, monotonically increase in τ (panel (b)).[15] Concentration is very high and above the social planner's preferred level, if $\tau \geq 0.5$.

TS^{PDV} also follows the monotonically increasing-then-monotonically decreasing in τ, predicted by part 4 of Proposition 3 (panel (c)). Efficiency is maximized for $\tau \approx 0.25$, reflecting the D_1^*s being quite close to their socially optimal levels. The right axis shows the values of the Besanko, Doraszelski, and Kryukov (2019a) PR and MS distortions.[16] DWL_β^{PR} is negative when $\tau = 1$ because, as discussed above, myopic buyers maximize current surplus in this case, whereas the social planner sacrifices surplus in asymmetric states to move the industry towards states where surplus is higher. This leads DWL_β^{MS} to be positive. The signs of these distortions are flipped when $\tau = 0$.

Panel (d) breaks down TS^{PDV} into buyer (CS^{PDV}) and seller (PS^{PDV}) surplus. As illustrated in Appendix Figure C.2, in a one-shot bargaining game with fixed production costs, expected producer and buyer surplus often change close to linearly in τ.[17] In contrast, in the dynamic game where costs can change, the gradients of the surplus curves are much larger, in absolute value, for small τ, and CS^{PDV} is actually non-monotonic.

Prices and Dynamic Incentive Comparative Statics. Figure 3(a) and (b) show prices, which determine the changes in the D_1^*s when buyers are myopic. In contrast to what happens in a static model (part 2 of Proposition 1), equilibrium prices can move up or down with τ in

[15]Concentration increases because the increase in $D_1^*(2,3)$ for low τ is more than offset by the declining probability that state (2,3) is reached.

[16] Formally,

$$DWL_\beta^{PR} = \sum_{t=1}^{\infty} \beta^t \sum_{\mathbf{e}} \lambda_t(\mathbf{e}) \left[TS^{SP}(\mathbf{e}) - TS(\mathbf{e})\right]$$

and

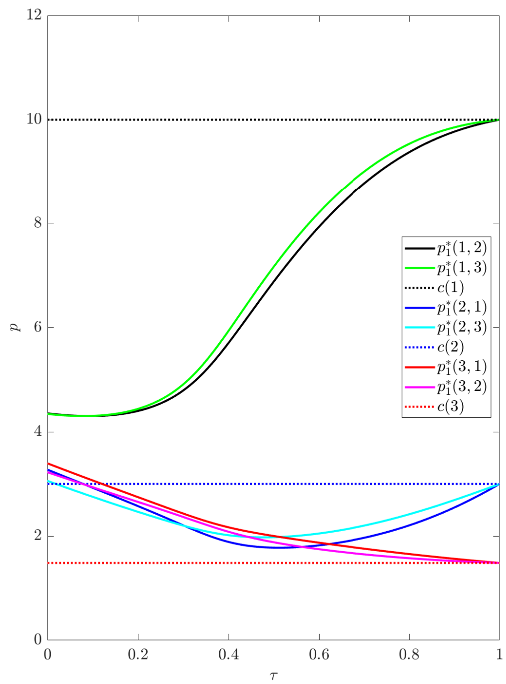
$$DWL_\beta^{MS} = \sum_{t=1}^{\infty} \beta^t \sum_{\mathbf{e}} \left[\lambda_t^{SP}(\mathbf{e}) - \lambda_t(\mathbf{e})\right] TS^{SP}(\mathbf{e}).$$

where $TS(\mathbf{e})$ and $TS^{SP}(\mathbf{e})$ represent the value of expected εs less production costs given equilibrium and social planner choice probabilities in state $\mathbf{e}$ respectively, and $\lambda_t(\mathbf{e})$ and $\lambda_t^{SP}(\mathbf{e})$ are the probabilities the game is in state $\mathbf{e}$ after t periods. $DWL_\beta^{PR} + DWL_\beta^{MS}$ is the difference between social planner and equilibrium surplus. Besanko, Doraszelski, and Kryukov (2019a) also consider a third, extensive margin, distortion related to entry and exit. The extensive margin is not relevant in the BDKS model.

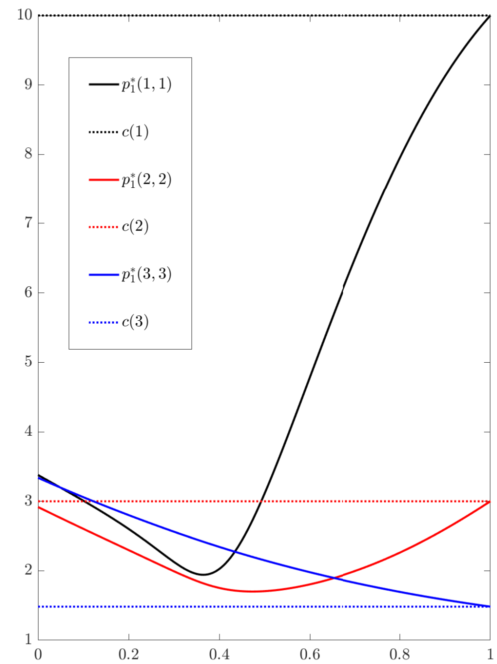
[17]Of course, the exact shape depends on the form of demand and the difference in sellers' costs. The changes in surplus become smaller as sale probabilities approach 0 or 1 with logit demand.

Figure 3: Equilibrium Prices, Dynamic Incentives and Lead Lengths as a Function of τ for $M = 3$ model with $\rho = 0.3$ and $\delta = 0.03$. SP = social planner outcome. DWL = deadweight loss. $c(e_i)$ are production costs for a firm with know-how e_i.

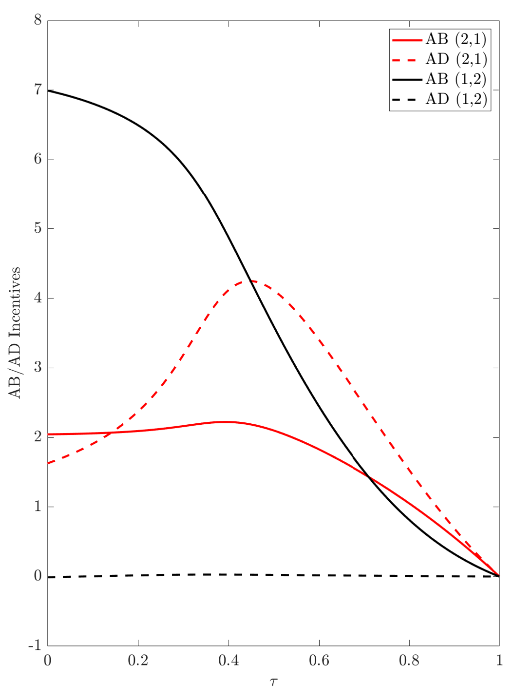
(a) Prices in Asymmetric States.



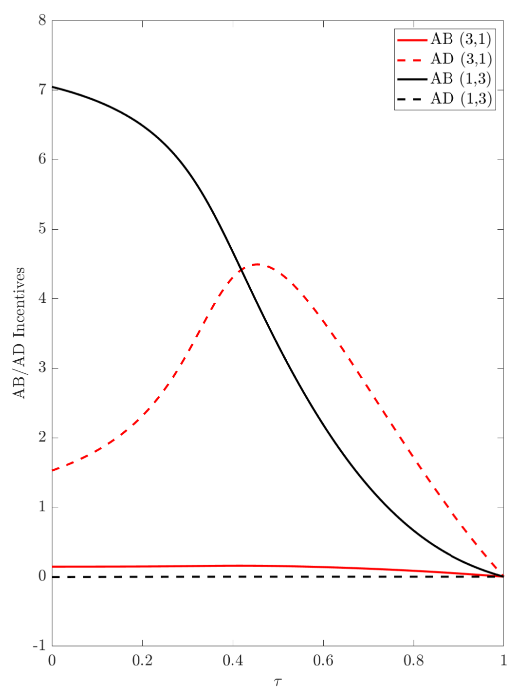
(b) Prices in Symmetric States.



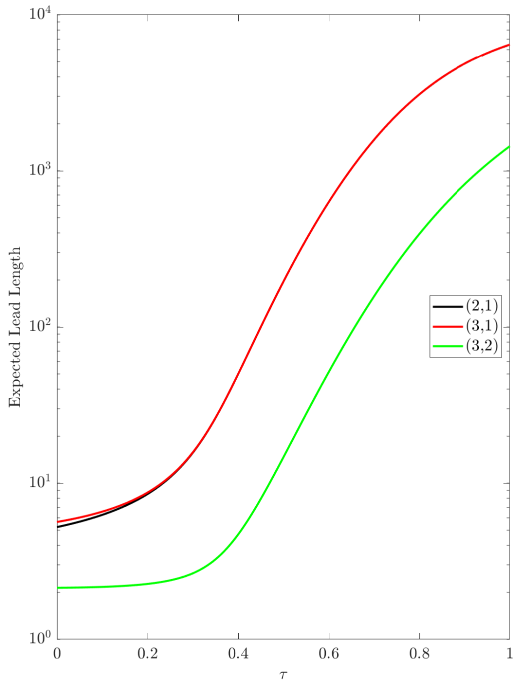
(c) Dynamic Incentives in $\mathbf{e}$ =(2,1)



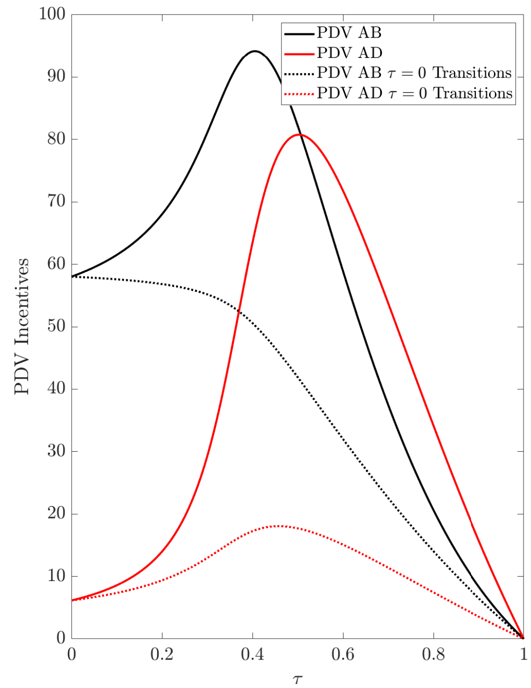
(d) Dynamic Incentives in $\mathbf{e}$ =(3,1)



(e) Number of Periods Lead is Expected to Last.



(f) PDV Dynamic Incentives



the dynamic model.[18]

The non-monotonicities in prices therefore reflect how dynamic incentives change. Panels (c) and (d) plot the AB and AD incentives of both firms in states (2,1) and (3,1). The value of both incentives must go to zero as τ approaches 1, as sellers' get less and less surplus in all negotiations, but some incentives, including AD(2,1), AD(3,1) and AB(2,1), increase with τ for $\tau < 0.5$, which will cause prices to decline for sellers in these states.

To understand what is happening, consider state (2,1). If the leader makes the sale, the state is likely to move to (3,1), or else will remain at (2,1), in the next period. If it loses the sale, the state will likely be (2,2). Because $D_1^*(1,3)$ and $D_1^*(1,2)$ are both relatively high when $\tau = 0$, the laggard rival is likely to catch up soon in any event, leading to relatively small dynamic incentives for the leader. As τ increases, $D_1^*(1,3)$ and $D_1^*(1,2)$ fall, partly reflecting how the leader's margin shrinks more than the laggard's margin (parts 2 of Proposition 1 and Corollary 1). The (2,1) leader can therefore expect to retain its lead for longer if it

[18] Cabral and Riordan (1994) associate single firm dominance with prices satisfying properties of "increasing dominance" (ID, lower cost firms always set lower prices) and "increasing increasing dominance" (IID, $p_1^*(\mathbf{e}) - p_2^*(\mathbf{e})$ decreases in e_1). Consistent with an observation of BDKS, we find that concentration can be socially excessive even when neither of these properties hold (e.g. $0.25 < \tau < 0.445$).

makes the sale and its rival does not, which will tend to increase its dynamic incentives. Assuming leads are profitable, this lead lengthening effect can dominate the declining share of surplus in each negotiation when leads do not last too long.[19]

Figure 3(e) complements this discussion by showing how τ affects the equilibrium number of periods that a lead is expected to last in all of the asymmetric states (the (2,1) and (3,1) lines essentially overlap for $\tau \geq 0.3$). Moving from $\tau = 0$ to $\tau = 0.5$ can increase how long leads last by a factor of more than 10. The solid line in Figure 3(f) shows the PDVs of dynamic incentives given equilibrium play. In this case, both the AB and AD lines increase sharply, by large absolute amounts, with τ before decreasing. The dotted lines show the PDVs calculated using $\tau = 0$ equilibrium λ weights. The differences between the solid and dotted lines therefore illustrate that, as τ increases, play shifts towards states, typically asymmetric states, where dynamic incentives tend to be larger.[20]

Generalization to Other Technology Parameters. Our $M = 3$ example does not show that these outcome and incentive comparative statics hold generally. Appendix D provides an analysis for the $M = 30$ example ($\rho = 0.75$, $\delta = 0.023$) that we will consider in applications below, and Appendix C.3 provides an analysis for $M = 3$ and technology parameters where multiple equilibria exist, and where, partly as a result of the multiplicity, the comparative statics are somewhat different. Here we focus on trying to provide a high-level sense of the comparative statics across technology parameters for the $M = 3$ and $M = 30$ models using the equilibria identified in our homotopy analysis. If there are multiple equilibria for a given τ, we take the maximum value of the statistic across equilibria. We then make comparisons across the 21 values of τ in 0.05 increments, acknowledging that this means we may miss small non-monotonicities occurring between the considered τs.

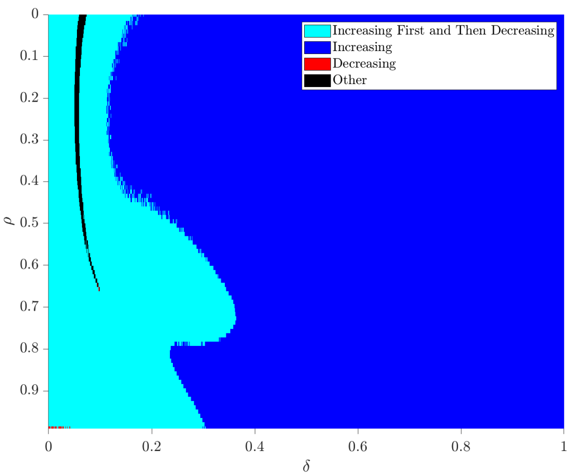
Figure 4 presents the results, for the full range of technologies for $M = 3$ and the restricted range for $M = 30$. Panels (a) and (b) summarize the monotonicity of TS^{PDV} with

[19] As the assumed value of δ implies that the leader's know-how is unlikely to depreciate if it does not make a sale, the critical issue in preserving a lead is that the laggard rival does not make a sale, as doing so would facilitate the laggard catching up. This provides the intuitive explanation for why, for these parameters, the most pronounced non-monotonicities are for AD incentives.

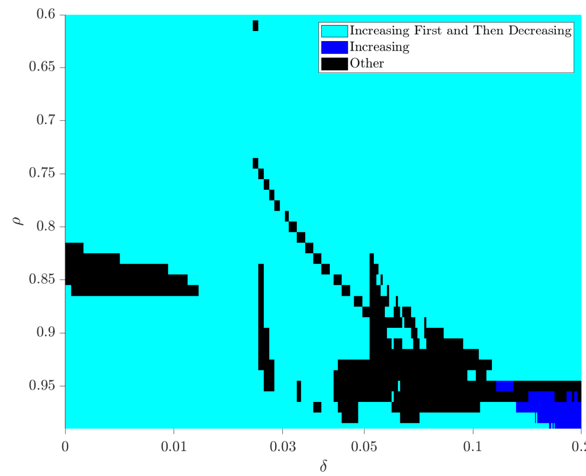
[20] For example, both AB and AD incentives are less than 0.17 for all τ in state (3,3), which will be the state after 32 periods with probability greater than 0.82 if $\tau = 0$, probability equal to 0.10 if $\tau = 0.5$ and probabilities less than 0.003 as τ approaches 1.

Figure 4: Comparative Statics and Dynamic Incentives for the $M = 3$ and $M = 30$ models for Different Technologies. We take the maximum value of the statistic across equilibria when multiple equilibria exist for given τ.

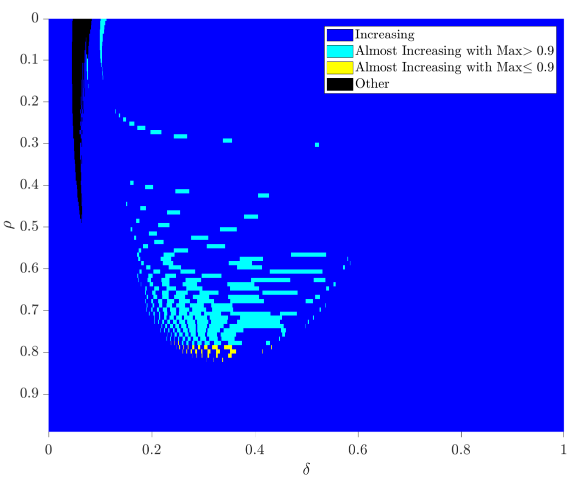
(a) $M = 3$: Monotonicity of TS^{PDV}



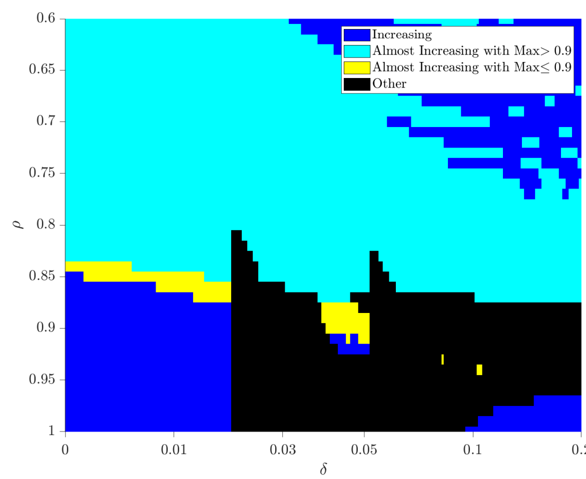
(b) $M = 30$: Montonicity of TS^{PDV}



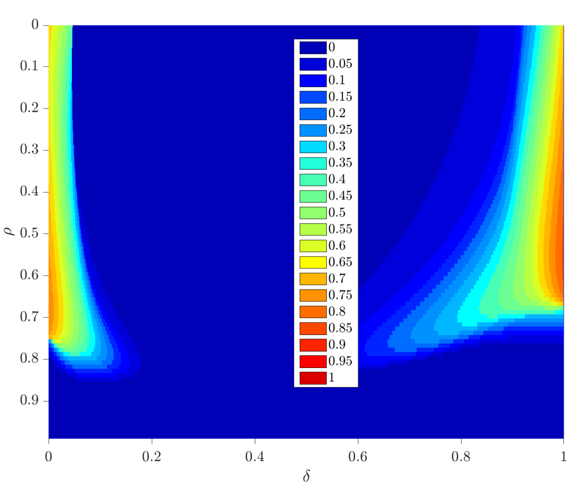
(c) $M = 3$: Montonicity of HHI^{32}



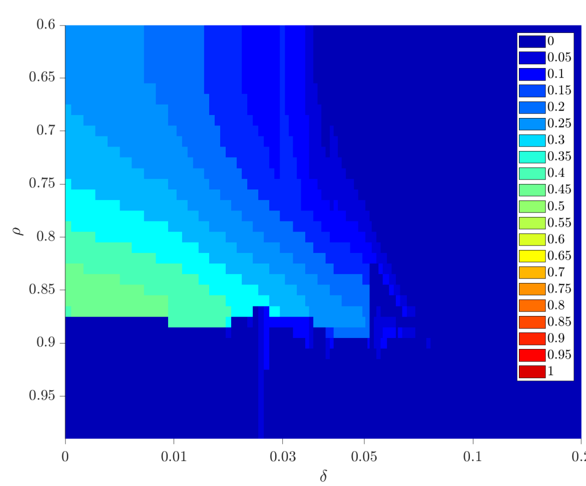
(d) $M = 30$: Montonicity of HHI^{32}



(e) $M = 3$: τ Maximizing PDV Dynamic Incentives



(f) $M = 30$: τ Maximizing PDV Dynamic Incentives



respect to τ. The light blue indicates parameters that correspond to the pattern observed above: TS^{PDV} monotonically increases up to some τ' and then monotonically decreases. The dark blue and the red colors (red only appears for a tiny area where ρ is close to 1 in the $M = 3$ figure) indicate monotonically increasing and monotonically decreasing efficiency respectively, while black indicates parameters which are not classified in the above groups.

For $M = 3$, monotonically increasing and then monotonically decreasing is the clearly dominant pattern for $\delta < 0.15$ for all ρ. Comparing with Figure 1(a), the exceptions are associated with technologies that support, or are close to parameters that support, multiple equilibria for $\tau = 0$. For higher δs, TS^{PDV} increases montonically in τ, reflecting how, when depreciation is likely, the social planner will prefer to concentrate sales on a single seller to guarantee that one seller has low production costs. When $\tau = 1$, the equilibrium with myopic buyers also tends to achieve this type of outcome.

For $M = 30$, where we are only looking at $\delta < 0.2$, monotonically increasing-then-monotonically decreasing TS^{PDV} is also the typical pattern when equilibria are unqiue, although there is a range of parameters with low δ, and ρs around 0.825, where equilibria are unique but this pattern fails. A closer look finds that, for these parameters, the increasing-then-decreasing pattern holds until τ is high (e.g. 0.8 or higher), at which point TS^{PDV} may increase by very small amounts. Appendix Figure D.4 presents two examples.

Panels (c) and (d) show the patterns for HHI^{32}. In our $M = 3$ example, HHI^{32} increases monotonically with τ. For other parameters, especially in the $M = 30$ model, a common pattern is that concentration increases monotonically and by a large amount, up to some τ', before decreasing, by a small amount, as τ increases to 1. Appendix D.2 provides an example. This "overshooting" can be explained by how a leader's dynamic incentives will tend to increase the leader's sales and concentration, and the fact that these incentives must disappear as τ increases 1. The light blue and yellow colors indicate technology parameters where concentration follows this pattern and the decline in HHI^{32} from its peak value to its value when $\tau = 1$ is less than 0.025. For almost all technology parameters, excepting some parameters where there are multiple equilibria, HHI^{32} is either monotonically increasing in τ or it meets these "almost increasing" criteria.

We also consider sellers' dynamic incentives. As we already know that τ changes different incentives in different ways, panels (e) and (f) indicate the τ value for which the PDV of combined dynamic incentives is maximized rather than trying to classify based on monotonicity. For ρs that imply significant LBD, values of τ between 0.2 and 0.8 maximize dynamic incentives as long as δ is not too high for both the $M = 3$ and $M = 30$ models. This is also true if depreciation is very likely in the $M = 3$ model, although such parameters seem unlikely to be relevant empirically. If LBD is limited, dynamic incentives are maximized when sellers set prices.

5 Extensions and Applications

So far we have only changed BDKS's assumption about how prices are determined. We move further beyond BDKS's assumptions in this section, in order to understand the robustness of these conclusions and the role that the bargaining weight could play in affecting predictions in stylized, but substantive, applications.[21] We primarily focus on our $M = 3$ and $M = 30$ example parameters where LBD is significant and depreciation is not too likely. Appendix D.3 gives some results for parameters with limited LBD, but more exhaustive examinations are left to future work.

5.1 Robustness: Forward-Looking Buyers.

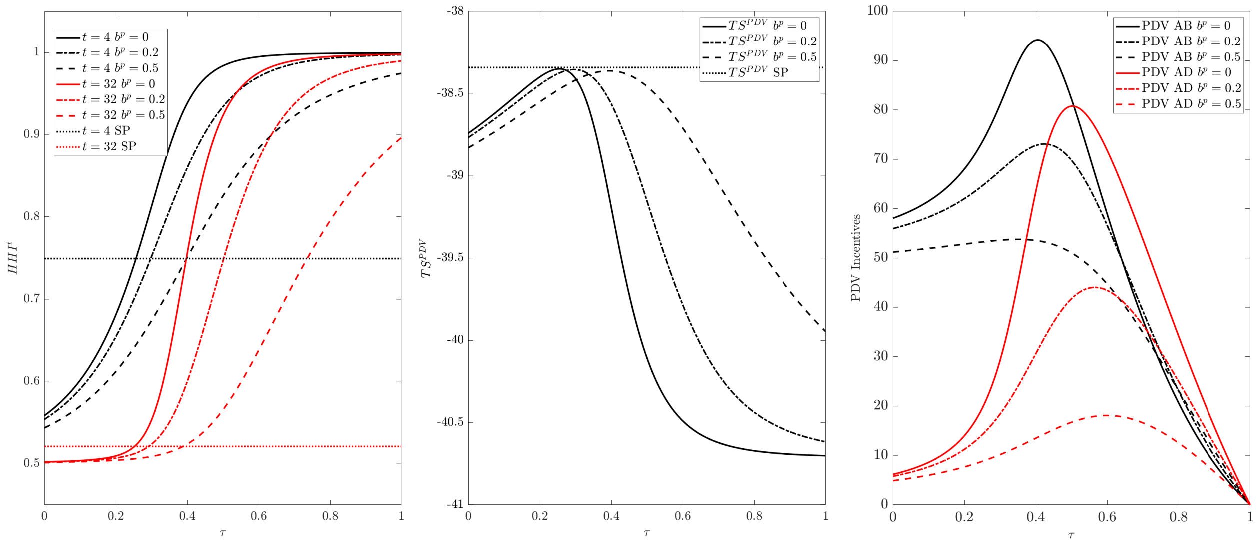
The assumption that buyers do not internalize how their choices impact future market structure is an important source of inefficiency in the BDKS model. However, in several relevant industries where prices are set by negotiation, buyers are also likely to internalize some of these effects because they may be in the market repeatedly.

SJHY provide a tractable representation of forward-looking buyers, summarized in Appendix E.2, by assuming that each buyer internalizes a proportion b^p of their impact on future buyer surplus. This can be motivated by assuming a pool of $\frac{1}{b^p}$ symmetric and infinitely-lived potential buyers, with nature choosing one of them, with replacement, to be the active buyer in each period. Assuming seller price-setting and the related BDK model, SJHY find that

[21] The text considers the effects of allowing for forward-looking buyers and more sellers. Appendix E.1 considers introducing an outside good and varying product differentiation in the duopoly seller and myopic buyer model.

Figure 5: Concentration, Welfare and Dynamic Incentives as a Function of τ for $M = 3$ model with $\rho = 0.3$ and $\delta = 0.03$, and Forward-Looking Buyers.

(a) HHI^t (b) TS^{PDV} (c) PDV Dynamic Incentives



forward-looking buyers tend to favor buying from laggards, which softens competition by reducing sellers' incentives to gain a cost advantage.[22]

Figure 5 shows what happens, in our $M = 3$ example, to the relationships between τ and HHI^{32}, TS^{PDV} and dynamic incentives when $b^p = 0.2$ (5 buyers) and $b^p = 0.5$ (2 buyers), as well as the results when $b^p = 0$ (our baseline) for comparison. Consistent with SJHY's findings, concentration and sellers' dynamic incentives (for $\tau < 1$) decrease as b^p rises. However, even though $b^p = 0.5$ assumes as much concentration on the buyer side as the seller side, the qualitative relationships between τ and the three outcome measures remain the same.[23] Efficiency is maximized at very close to the social planner's level in all three cases. One way to think about what is happening is that, as τ increases, forward-looking buyers may have less incentive to try to promote price competition by making sellers symmetric as bargaining power gives them the ability to extract more surplus from a seller that has a significant cost advantage.

[22] SJHY also show that multiple equilibria are eliminated once b^p increases beyond 0.2 in both the BDK and the BDKS models. We find that multiplicity is eliminated for even lower b^ps if $\tau > 0$ in our extended BDKS model.

[23] Several equilibrium prices (not shown), including (1,1), (2,2) and (2,1), also continue to be non-monotonic (decreasing, then increasing) in τ for both b^ps.

Appendix Figure E.3 shows the same set of diagrams for our $M = 30$ example (i.e. $m = 15$, $\rho = 0.75$ and $\delta = 0.023$).[24] The patterns are broadly similar to those in Figure 5, although when $b^p = 0.5$, HHI^4 does not quite reach the socially optimal level even when $\tau = 1$.[25]

5.2 Robustness: Additional Sellers.

We relax the duopoly assumption by allowing for three or four sellers. To maintain tractability, we assume $M = 7$ and $m = 5$. We set $\rho = 0.65$, implying a minimum marginal cost of 3.66, comparable to our other examples, and $\delta = 0.03$.

Figure 6 shows equilibrium HHI^7, HHI^{32}, TS^{PDV} and the PDVs of dynamic incentives, as functions of τ, for different numbers of sellers. Adding sellers allows lower concentration, but, concentration increases in τ for each N, apart from some slight overshooting for HHI^7. TS^{PDV} maintains its increasing-then-decreasing relationship with τ, with maximums quite close to the social planner level when τ is between 0.3 and 0.4.[26] While the patterns for dynamic incentives vary slightly across N, and peak AD incentives become smaller as N increases, AD incentives are maximized for $\tau \approx 0.6$ in each case, and AB incentives display some form of non-monotonicity. Therefore, the comparative statics of outcomes with respect to τ in the oligopoly model with up to four sellers are broadly similar to the comparative statics observed for many technology parameters in our duopoly models.

5.3 Application: Optimal Subsidies.

Our first application considers subsidy schemes that implement the social planner outcome as an equilibrium by making state-specific transfers to, or from, a laggard making a sale.[27]

Given socially optimal choice probabilities (D_1^{SP}), we can solve for M^2 prices and $\frac{M(M-1)}{2}$

[24]While $t = 4$ represents the very short-run for an $M = 30$ model, we still show it partly because, when $b^p = 0$, this figure provides a clear illustration of the HHI^t overshooting effect with respect to τ.

[25]If $b^p = \tau = 1$, then the social planner's solution is implemented.

[26]For these parameters, efficiency increases with N for all τ, although we have seen examples where this is not true for all τ.

[27]With no outside good, sale probabilities in symmetric states are efficient so it is natural to restrict attention to transfers in asymmetric states.

Figure 6: Concentration, Total Surplus and Dynamic Incentives as a Function of τ when $M = 7$, $m = 5$, $\rho = 0.65$, $\delta = 0.03$ and $N = 2$ to $N = 4$.

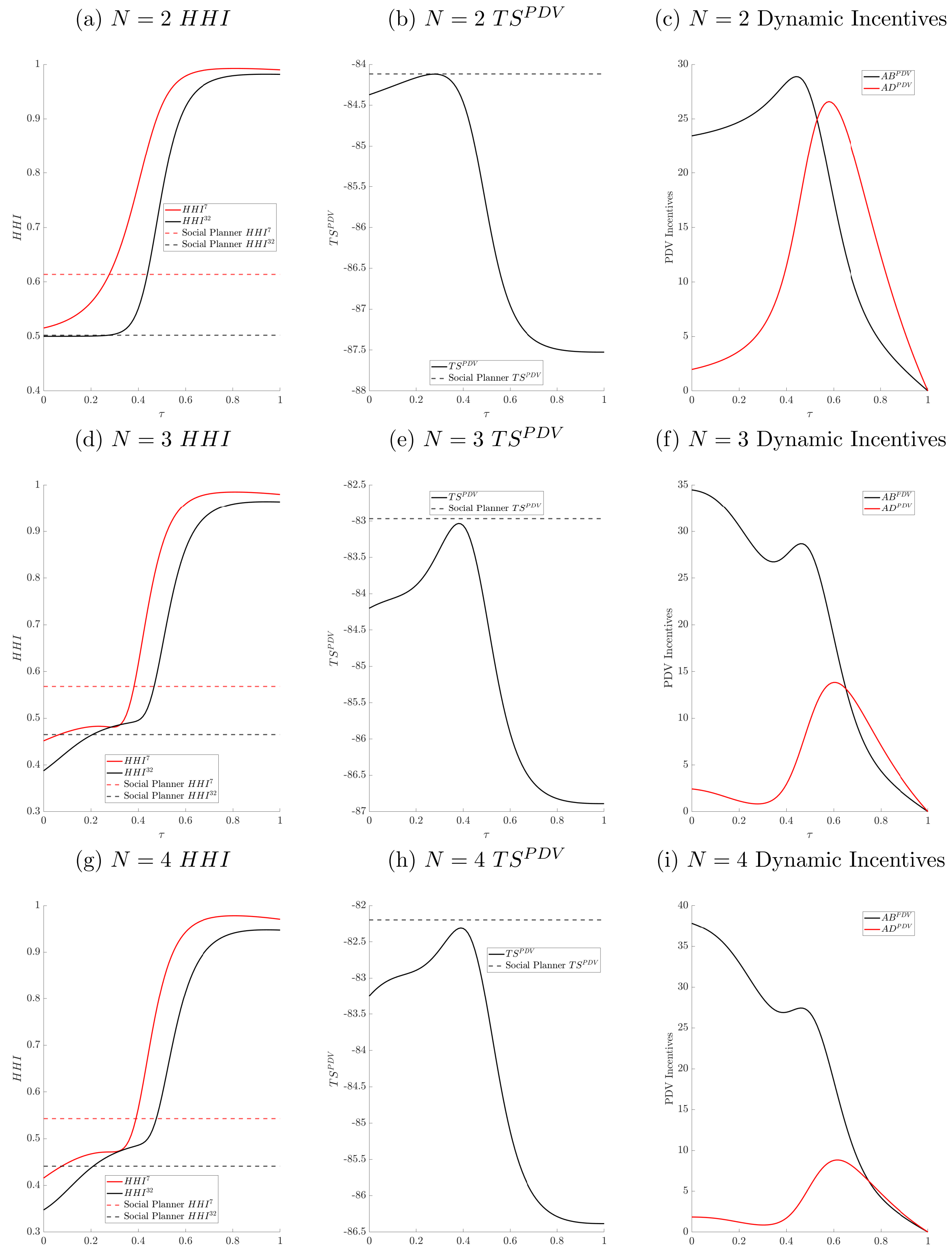
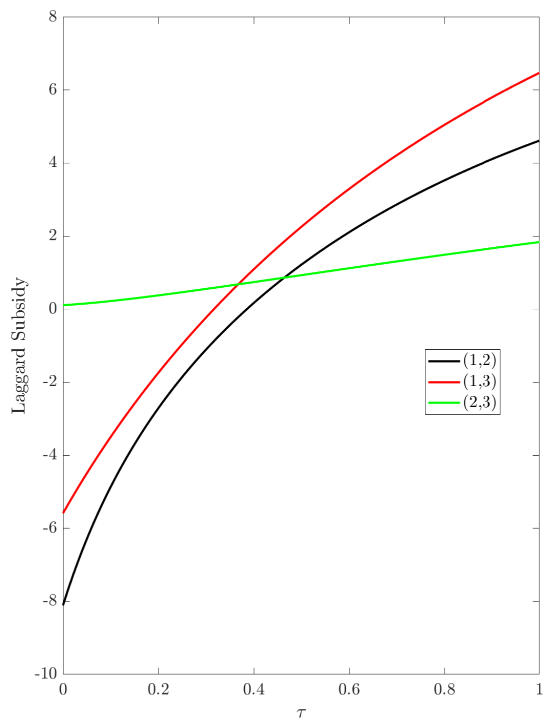
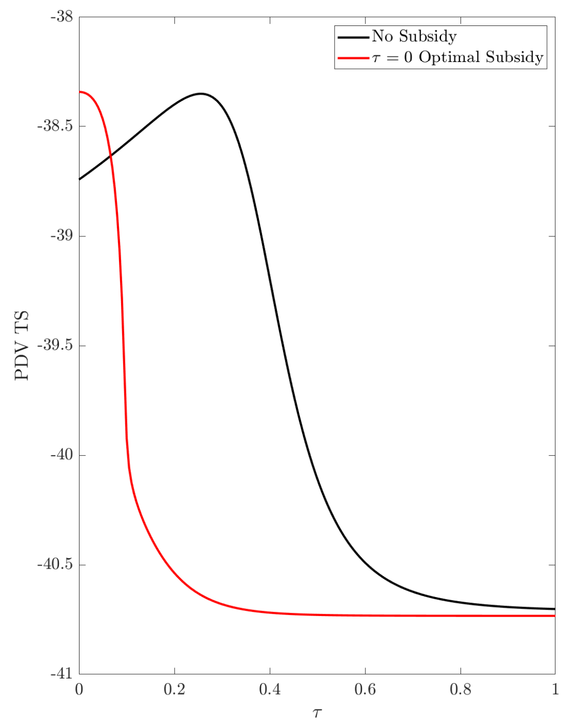


Figure 7: Optimal Subsidy Schemes and Welfare as Functions of τ for the $M = 3$ and $M = 30$ examples. Negative subsidies are taxes on the laggard when it makes a sale.

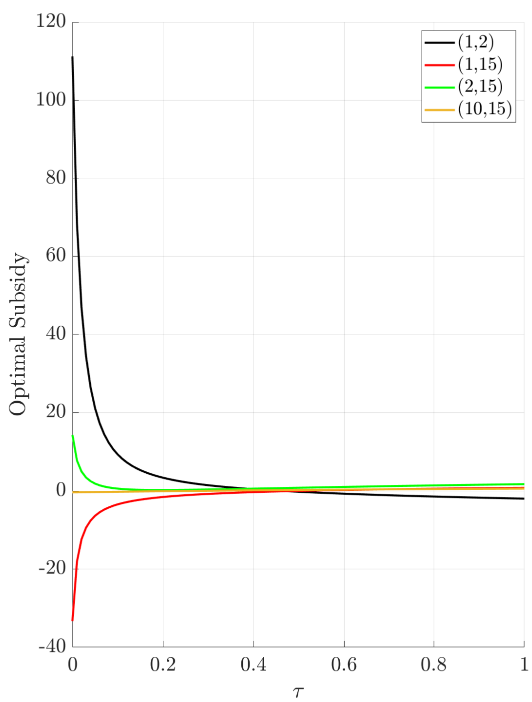
(a) $M = 3$: Optimal Subsidy Levels



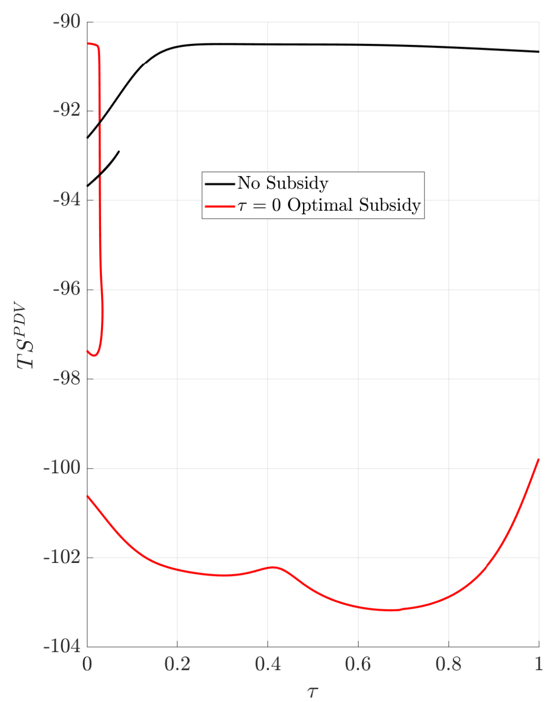
(b) $M = 3$: TS^{PDV} with $\tau = 0$ Optimal Subsidies



(c) $M = 30$: Optimal Subsidy Levels in Selected States



(d) $M = 30$: TS^{PDV} with $\tau = 0$ Optimal Subsidies



subsidies (s_1, negative for a tax) using $\frac{M(M-1)}{2}$ equations

$$p_1(\mathbf{e}) - p_2(\mathbf{e}) = \sigma \log \left(\frac{1}{D_1^{SP}(\mathbf{e})} - 1 \right), \tag{17}$$

and M^2 equations

$$\mathbf{p_1} + \mathbf{s_1} = \mathbf{\Phi}(\mathbf{D_1^{SP}}) + \mathbf{c_1} - \beta(\mathbf{Q_1} - \mathbf{Q_2})(\mathbf{I} - \beta\mathbf{Q_2})^{-1}[\mathbf{D_1^{SP}} \circ \mathbf{\Phi}(\mathbf{D_1^{SP}})]. \tag{18}$$

The linearity of (18) in p and s implies that a unique scheme can implement the social optimum, although this scheme may also support sub-optimal equilibria. We calculate optimal subsidies as functions of τ for our $M = 3$ and $M = 30$ examples. As shown in Section 4.4 and Appendix D.2, concentration is too low when $\tau = 0$ and too high when $\tau = 1$ in both cases.

Figure 7(a) shows the optimal transfers for $M = 3$. Laggard sales are taxed in states $(1, 2)$ and $(1, 3)$ when $\tau = 0$, but they are subsidized in all states when $\tau > 0.4$. The direction of these transfers are intuitively consistent with the laggard being too likely (unlikely) to make sales when τ is small (large) in the no subsidy equilibria. For $M = 30$ (panel (c)) some optimal transfers decrease dramatically in absolute value when we move away from price-setting.

The directions of some of the $M = 30$ transfers may also appear counterintuitive, but they reflect the role of dynamic incentives. When the social planner's evolution is implemented, a leader in a state such as (3,1), will expect its lead to last a very long time (798 periods, versus 88.5 periods with the equilibrium transition), which, when τ is small, will cause the leader to have very large dynamic incentives in a preceding state such as (2,1). These incentives would cause the leader to be much more likely to make a sale than the social planner would like, so that a large subsidy to the laggard is required.

Motivated by the existing literature's assumptions, panels (b) and (d) show TS^{PDV} under subsidies that would be recommended by an analyst who assumed that sellers set prices ($\tau = 0$). In the $M = 3$ example, outcomes with these subsidies are worse than outcomes with no subsidies if $\tau \geq 0.06$. In the $M = 30$ example, the $\tau = 0$ optimal scheme, and no scheme, support multiple equilibria when τ is small. Depending on the equilibrium that is played, the subsidy scheme could actually lower efficiency even if $\tau = 0$. However, the $\tau = 0$ scheme unambiguously lowers welfare, relative to no scheme, if $\tau \geq 0.04$. Therefore, designing policies that assume seller price-setting could be damaging even when buyers' bargaining power is very limited.

The appendices contain some additional analysis of subsidies. Appendix D.3 shows $\tau = 0$ subsidies can lower TS^{PDV} for small values of τ that are greater than zero even when LBD is limited ($\rho = 0.95$). As a comparison to these dynamic model results, Appendix E.4.1 performs the same exercises but assuming static seller behavior, finding that, in this case, $\tau = 0$ optimal subsidies would improve welfare for all τ in our $M = 30$ example, and for $\tau < 0.35$ in the $M = 3$ example. Appendix E.4.2 tries to repeat the $M = 3$ model dynamic analysis for alternative technology parameters. When δ is small, $\tau = 0$ optimal subsidies generally lower welfare, relative to no subsidies, if $\tau = 0.2$. We also show that optimal transfers can be extremely large when δ increases, leading to numerical issues that prevent welfare comparisons.

5.4 Application: Profitability and Efficiency of a Horizontal Merger.

Agency decisions when reviewing mergers in industries with LBD, such as Boeing's acquisition of McDonnell Douglas (a horizontal merger) or NVIDIA's proposed acquisition of ARM (a vertical merger), are often controversial because it is widely understood that standard analyses, based on assuming static seller behavior, are likely to miss important incentives and medium-run/long-run effects. The literature that explicitly analyzes mergers using dynamic models is small (e.g., Gowrisankaran (1999), An and Zhao (2019), Mermelstein, Nocke, Satterthwaite, and Whinston (2020)) and has not considered how predictions would change if prices were determined by bargaining, despite the attention that has recently been paid to incorporating bargaining into static models of mergers (Ho and Lee (2017), Gowrisankaran, Nevo, and Town (2015), Sheu and Taragin (2021)).

We consider the profitability and welfare effects of a horizontal merger using the $M = 7$, $m = 5$, $N = 4$, $\rho = 0.65$ and $\delta = 0.03$ parameterization from Section 5.2. We assume that the proposed merger is between the two leaders in state (1,1,3,3), that the merger will generate no cost (in)efficiency, or spillovers in know-how or forgetting across the merged firm's products, and that there will be no subsequent mergers. After a merger, we allow for the merged firm to either keep both products and jointly negotiate their prices with buyers[28], or to immediately eliminate one of the products. The following discussion assumes that there are no fixed costs.

If firms' production costs are permanently fixed, implying no dynamic incentives, then straightforward calculations show that, if $\tau < 1$, the merged firm would prefer to keep both products, that the merger is profitable whether or not both products are maintained, and that buyer surplus and total surplus will decrease. Appendix E.3.1 shows that the merger that maintains both products is also profitable, and that mergers decrease total and buyer surplus for all τ when costs can evolve but we assume static seller behavior. We now examine whether these predictions still hold in our dynamic model, and how this is affected by the value of τ.

The blue (red) line in Figure 8(a) shows the value, i.e., the PDV of the merged firm's profits, starting from the moment that the merger is proposed, as a function of τ, if it continues to sell both products (eliminates one of its products). The black line shows the combined value of the parties if the merger is not consummated. The figure shows that, in the dynamic game, profitability comparisons depend on τ: for example, the parties' value is higher if they do not merge if $\tau \leq 0.35$. The merging parties' value is always higher when they keep both products, but merging and eliminating a product gives a higher value than not merging if and only if $\tau \geq 0.6$.

Panels (b) and (c) also show that, while, for all τ, a merger with a product elimination is the worst outcome for total surplus and for buyer welfare, a merger where both products are maintained increases total surplus if $0.35 \leq \tau \leq 0.5$ and buyer welfare if $0.14 \leq \tau < 0.35$.[29]

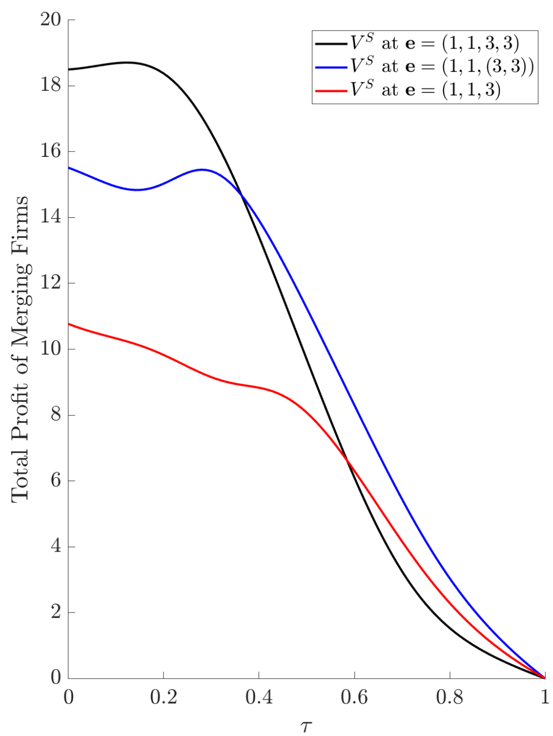
We can look more closely to explain these results. Figure 9 compares the dynamics of expected shares, the difference between expected prices and expected production costs (i.e., these are not the Φ markups over opportunity costs) and profits in the sixty periods after a merger is proposed, comparing no merger with a merger that maintains both products, if

[28] This means that if there is disagreement the buyer will be unable to buy either product. Gowrisankaran, Nevo, and Town (2015) consider this assumption and an alternative assumption that the merged firm has to negotiate the price of each product separately.

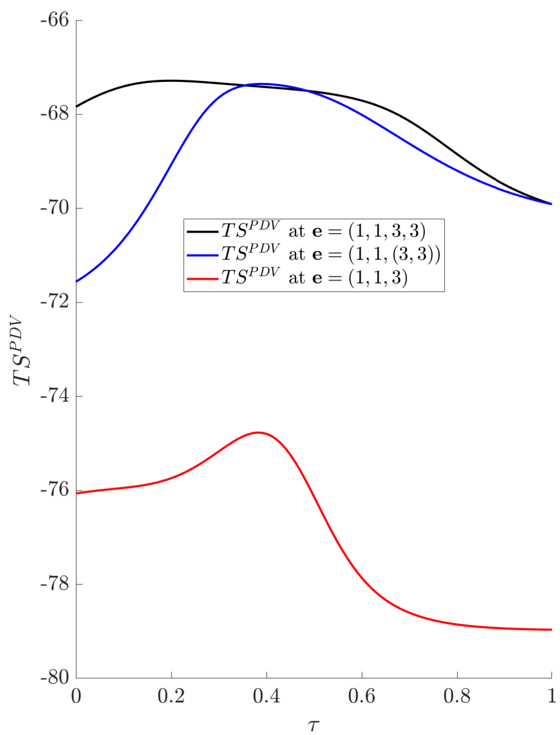
[29] In this example, there are no τs for which a profitable merger is socially efficient and good for buyers.

Figure 8: Effects of a Horizontal Merger Between Firms with $e_3 = 3$ and $e_4 = 3$, when Rivals have $e_1 = 1$ and $e_2 = 1$, as a Function of τ for $M = 7$, $m = 5$ for $\rho = 0.65$ and $\delta = 0.03$.

(a) Merging Firm Values



(b) TS^{PDV}



(c) CS^{PDV}

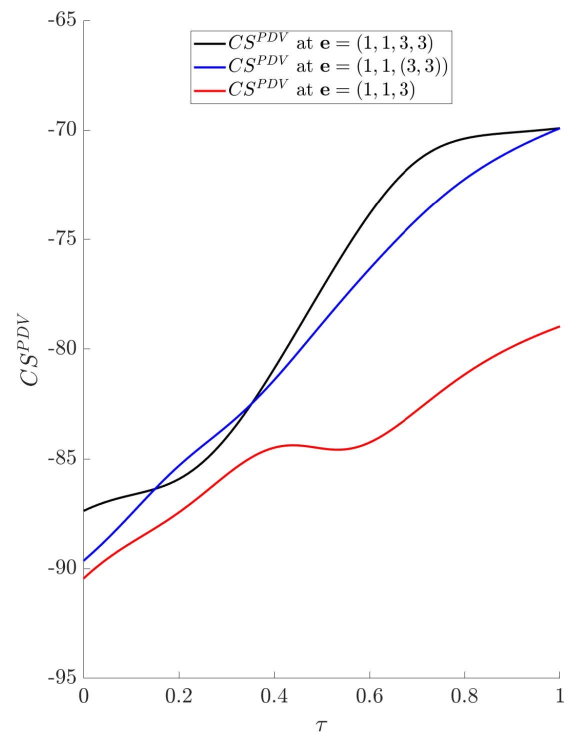
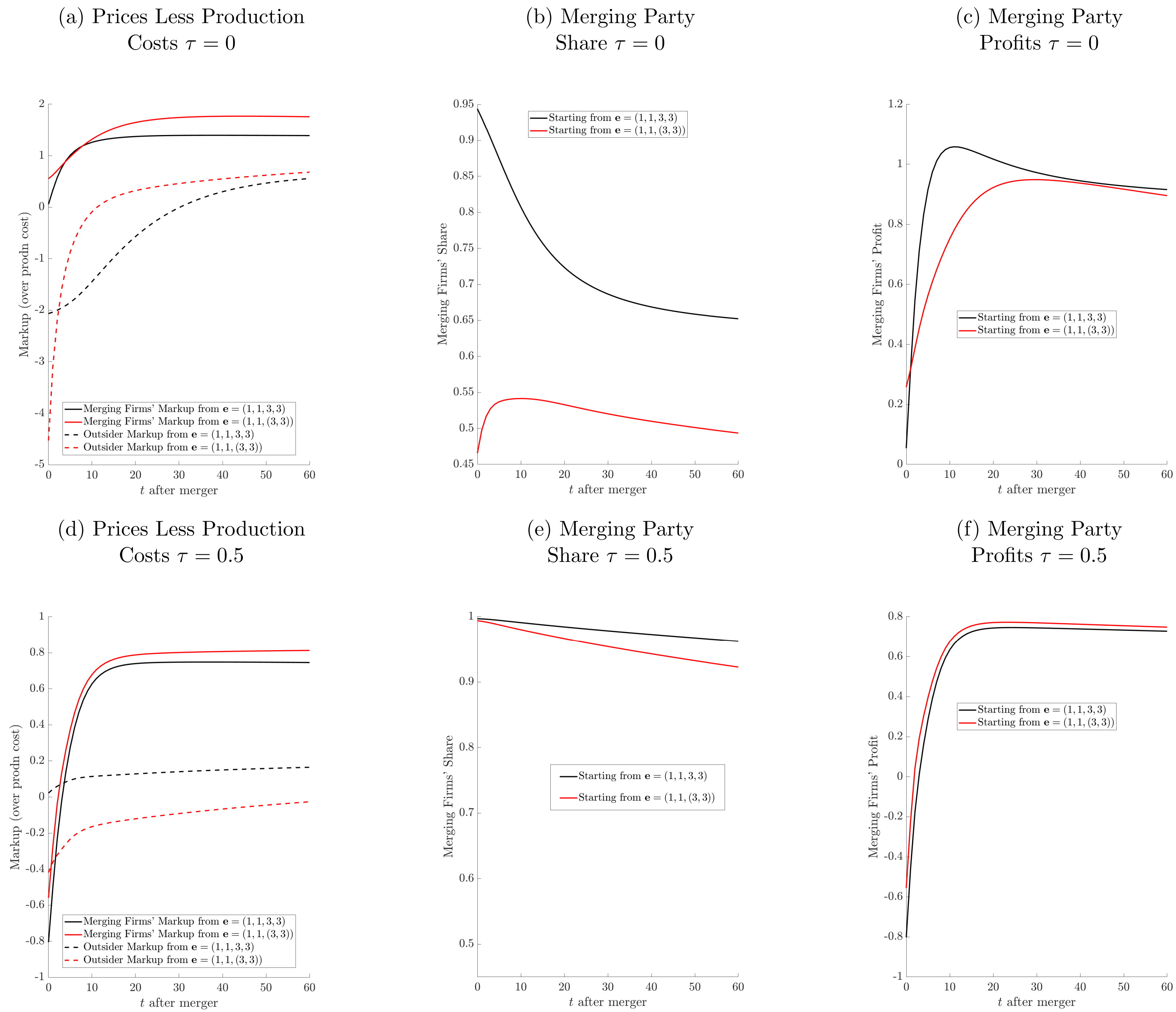


Figure 9: Comparison of Post-Proposed Merger Dynamics for $\tau = 0$ and $\tau = 0.5$, Assuming the Merged Firm Keeps Both Products.



$\tau = 0$ or $\tau = 0.5$. Recall that this merger is only profitable for the parties in the latter case.

If $\tau = 0$, the merger increases the parties' margin (over production costs) from almost zero to 0.6 in the first period. This is consistent with a market power effect, but also a drop in the parties' AD incentives from 0.2 to -0.27 (the parties' AB incentives are almost unchanged at 1.6). The equilibrium prices of the non-merging parties drop significantly, which, of course, would not happen in a static merger analysis with logit demand. This drop reflects the laggard firms anticipating that, post-merger, one of them will be able to quickly catch up, so that their dynamic incentives increase (specifically, their AB incentives increase from 3.08 to 5.58). The price changes cause the combined market share of the merging parties drops from 0.95 to less than 0.5. The industry then quickly evolves to states where there is one low cost multiproduct firm with high prices and one low cost single product firm with lower prices, with the latter taking around one half of sales. In contrast, with no merger, the leaders' are likely to remain ahead of the other firms as they move to the lowest cost states, and a third firm will then tend to catch up more slowly so that the parties' maintain higher profits for a much longer period of time, explaining why it is more profitable for the parties not to merge.

The dynamics are qualitatively different if $\tau = 0.5$: the parties' share declines only slowly after a merger, reflecting a much smaller increase in the parties' prices and a smaller decrease in the rivals' prices. Intuitively, the sellers understand that the buyers' bargaining power will constrain the merged firm's markups in future periods, as well as immediately after the merger, so that the merged firm will recognize that if it can preserve a lead, the lead is likely to persist. This leads to the merger actually increasing the parties' dynamic incentives immediately following the merger (for example, the parties' AD incentives are -0.007 with no merger and 4.06 with merger), while causing the dynamic incentives of the non-merging parties to increase by a smaller amount than if $\tau = 0$.

5.5 Application: Policies to Promote Competition.

Industries with LBD may come to be dominated by a single firm if it establishes a sufficiently large cost advantage. Governments may try to limit dominance for either efficiency or non-economic reasons using a variety of (non-subsidy) policies. In this section, we consider the effects on concentration and welfare of the following stylized policies, focusing on how our conclusions vary with the assumed value of τ, using our $M = 30$ example. We will focus on policies that restrict a firm that is a leader.[30]

Restrictions on Pricing. The first restriction involves a ban on a leader pricing below its current production cost, motivated by how below-cost pricing is often viewed as a necessary, but not sufficient, condition for pricing to be viewed as predatory.[31] We track equilibria using the homotopy method with an additional set of equations associated with Lagrangian constraints. An initial equilibrium is identified using an iterative guess-and-verify approach, where, at each step, we identify states where the constraints bind.

Restrictions on Pricing Incentives. Besanko, Doraszelski, and Kryukov (2014) and Besanko, Doraszelski, and Kryukov (2019b) analyze outcomes in the BDK model if firms are not allowed to consider certain dynamic incentives, motivated by more sophisticated benchmarks for predation.[32] Specifically we consider how equilibrium outcomes change when the leader

- is unable to consider dynamic incentives at all (i.e., the leader's opportunity cost equals its current production cost); or,
- is unable to consider AD incentives, but can consider AB incentives (the level of which may change due to the policy).

Restriction on Market Concentration. Benkard (2004), analyzing the wide-bodied commercial aircraft market, considers a counterfactual where a limit is imposed on the market share of the largest firm in a given quarter. While absolute restrictions on shares are not common, market share thresholds may play important roles in trade policies (e.g., Voluntary Export Restraints) or the determination of potential liability for certain types of alleged

[30] The restrictions do not apply in states where the firms are symmetric.

[31] For example, see the discussion in the Department of Justice 2008 report "Competition and Monopoly: Single-Firm Conduct Under Section 2 of the Sherman Act", `https://www.justice.gov/sites/default/files/atr/legacy/2009/05/11/236681.pdf`. Of course, the appropriate measure of costs is often disputed in litigation.

[32] Even though the BDKS model does not allow exit, the critical issue in predation analysis is really whether the leader's aggressive pricing, or the possibility of aggressive pricing, can lead to a laggard being uncompetitive for long enough that the leader can recoup any lost profits.

anticompetitive behavior. In our duopoly single-buyer-per-period model, we implement the share restriction as a soft constraint by assuming that the leader has to pay a compliance penalty of $\chi \times \max\{0, D_i - \psi\}^2$ whenever its sale probability is above a threshold ψ.[33] In the following we assume $\psi = 0.75$ and $\chi = 50$, and that concentration penalties do not reduce total surplus. $\chi = 50$ is large enough that the threshold is rarely breached in equilibrium. Appendix E.5 shows the effects of several alternative χs.

Trigger Policies. So far we have assumed that the policies apply to all leaders (e.g., a firm in state (2,1)). In practice, government agencies may only intervene once a leader is clearly established. As the social planner may want to invest in creating a second low cost provider once the first firm has low costs, one might also think that a policy that only restricts a leader once its know-how is sufficiently high could be optimal. We therefore compute equilibria under "trigger versions" of each of the policies described above, meaning that the policy will be introduced as soon as one firm reaches know-how state e' and will then apply to whichever firm is the leader in all subsequent periods. We assume players know the value of e' when the game begins.[34]

Multiplicity under the Restrictions. For our $M = 30$ example, we find some multiplicity for small τ under the concentration restriction and Leader $p \geq mc$ policies, as well as in the absence of any policy. However, outcomes across these equilibria are sufficiently similar that they do not really complicate our observations about the policies. For other technology parameters, we have also identified multiplicity under the incentive policies.

Effect of Policies Introduced in Period $t = 1$. Figure 10(a) and (b) show the values

[33] Incorporating this penalty, the first-order condition for the negotiated price becomes

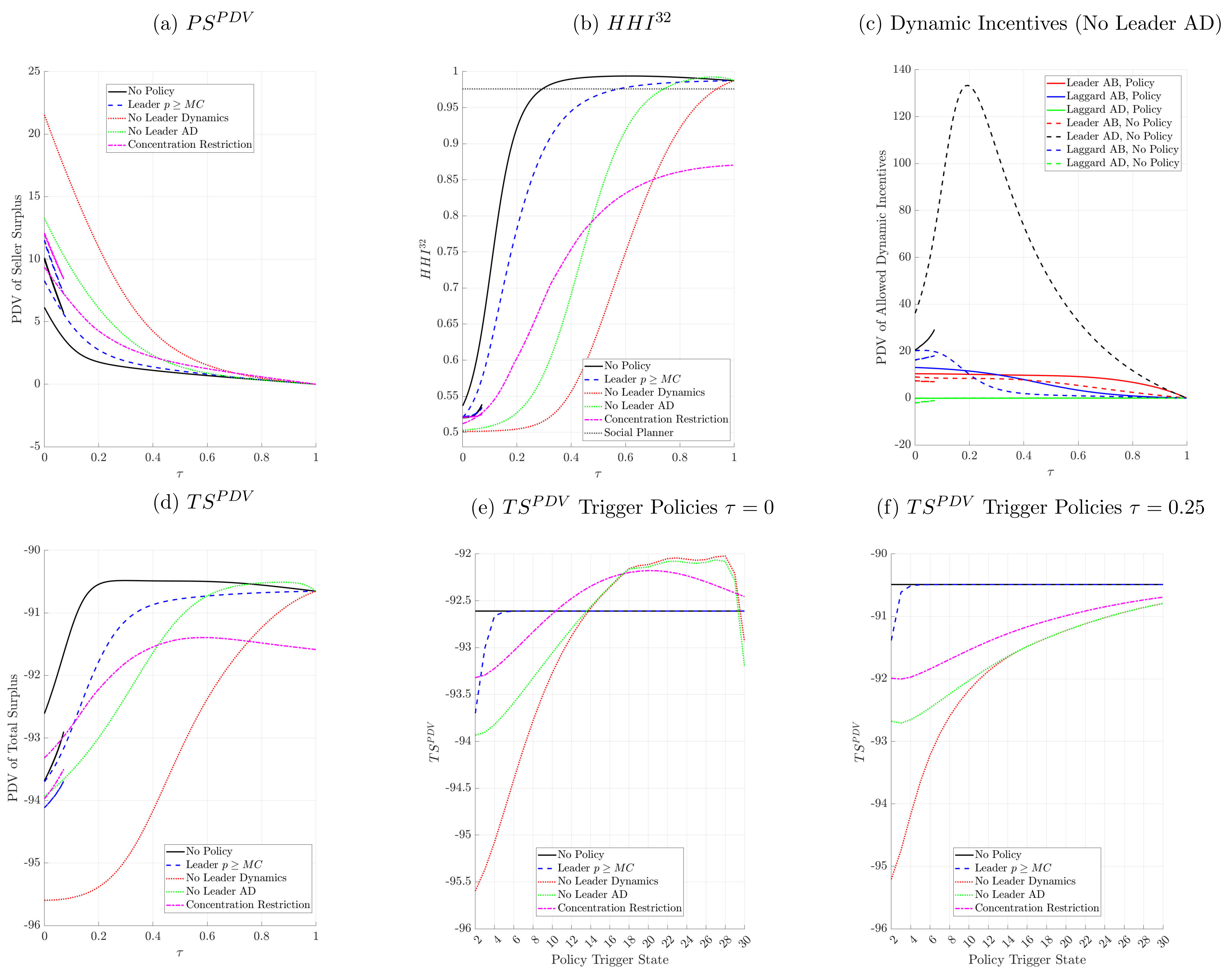
$$\begin{aligned} -\tau \left[D_1^*(\mathbf{e})(p_1^*(\mathbf{e}) - \widehat{c}_1) - \chi \max\{0, D_1^*(\mathbf{e}) - \psi\}^2\right] + (1-\tau)\log\left(\frac{1}{1 - D_1^*(\mathbf{e})}\right) \times \\ [\sigma - (1 - D_1^*(\mathbf{e}))(p_1^*(\mathbf{e}) - \widehat{c}_1) + 2\chi(1 - D_1^*(\mathbf{e}))\max\{0, D_1^*(\mathbf{e}) - \psi\}] = 0, \end{aligned} \tag{19}$$

and the equation for the seller's value becomes

$$V_1^{S,*}(\mathbf{e}) - D_1^*(\mathbf{e})\left(p_1^*(\mathbf{e}) - c(e_1)\right) - \sum_{k=1,2} D_k^*(\mathbf{e})\mu_{1,k}^S(\mathbf{e}) - \chi \max\{0, D_1^*(\mathbf{e}) - \psi\}^2 = 0. \tag{20}$$

[34] We solve a game with a trigger policy by first solving for strategies and values in a game where the restrictions apply, and then, for the chosen e', we solve a game with a smaller number of states where the appropriate firm values from the first game are the terminal values in states where the leader has state e'.

Figure 10: Outcomes under the Policies as a Function of τ for the $M = 30$ Example. Panels (a)-(d) assume policies introduced before play begins. The compliance costs of the Concentration Restriction policy are not counted as costs to society in total surplus calculations, although they are costs to the sellers.



of HHI^{32} and PS^{PDV}, as functions of τ, to illustrate the effects of the alternative policies on the firms and market structure. Panel (d) shows TS^{PDV}. Ignoring caveats associated with multiplicity, all of the policies raise PS^{PDV}, lower HHI^{32} and lower TS^{PDV} if $\tau \leq 0.76$ consistent with them softening dynamic competition.[35] The $p \geq mc$ restriction generally affects outcomes the least, consistent with no policy prices only being below costs in a relatively smaller number of asymmetric states.[36]

Contrary to the other results, the no leader AD incentive policy increases TS^{PDV} if $0.76 \leq \tau < 1$, lowers PS^{PDV} if $0.8 \leq \tau < 1$ and increases HHI^{32} if $0.9 \leq \tau < 1$.[37] Given that our discussion so far has highlighted how the leader's dynamic incentives tend to increase concentration, a pattern where concentration increases when the leader cannot consider a dynamic incentive may seem surprising.

The explanation is that dynamic incentives are, of course, endogenous. The dashed lines in panel (c) show incentives of the leader and the laggard (incentives in symmetric states, which only affect market structure indirectly are not shown) with no policy. The solid lines show allowed incentives under the no leader AD policy. For high τ, the leader's AB incentive (red line) rises with the policy, and, for high τ, it roughly equals the excluded leader's AD incentive.

Effect of Trigger Policies. Panels (e) and (f) show how trigger versions of the policies, with different trigger states, affect TS^{PDV} if $\tau = 0$ or $\tau = 0.25$.[38] The TS^{PDV} figure for $\tau = 0.5$, which is not shown, is very similar to the $\tau = 0.25$ figure, except that the concentration restriction policy curve falls relative to the no incentive policy curves.

An interesting result is that policies with appropriate triggers can increase efficiency when $\tau = 0$, but they do not do so if $\tau = 0.25$ (or $\tau = 0.5$). Further analysis indicates that the τ thresholds for the policies to reduce efficiency are really quite small: for example,

[35] Only the concentration restriction policy affects outcomes when $\tau = 1$, as it is only that restriction that can potentially be violated in a $\tau = 1$ no policy equilibrium. A leading seller is able to avoid violating it by not agreeing a price with the buyer.

[36] Online Appendix D.1 lists prices in a subset of states in the no policy $\tau = 0$ equilibria.

[37] One can put the TS^{PDV} and PS^{PDV} curves together to understand the effects on buyer surplus. Except for the no leader AD incentive policy for $0.76 \leq \tau < 1$, where there is a very small increase, all of the policies lower CS^{PDV} for all $\tau < 1$. The concentration restriction policy lowers CS^{PDV} for $\tau = 1$ as well.

[38] As none of the policies directly constrain sellers in state (1,1), the outcomes when the trigger is $e' = 2$ are identical to those when the policy is introduced from the start of the industry.

with triggers of $e' = 20$, either incentive policy or the concentration restriction policy lower TS^{PDV} if $\tau > 0.06$ or 0.08 respectively, providing a further illustration that conclusions from assuming seller price-setting could be misleading.

The direction of these TS^{PDV} effects when $\tau = 0$ may also seem paradoxical, in the sense that policies that reduce concentration are raising efficiency even though concentration with no policy is below the social planner's preferred level. The paradox is resolved by recognizing that the trigger policies can increase concentration in early periods of the game, as the laggard, anticipating that it will get assistance from the policy once the leader reaches the trigger threshold, has weaker incentives to try to catch-up.

5.6 Application: Exclusive Contracts and Foreclosure Shares.

A repeat buyer may plausibly want to sign long-run contracts, and a seller might also want to do so in order to guarantee some future sales. Our final application introduces such contracts in a stylized way, and considers their effects on competition for non-contracted customers.

Intuition suggests effects in different directions, suggesting the need for a model to weigh which effects dominate. For example, if one seller has contracts, the other seller may have higher costs, and, because it knows it will be at a disadvantage, a reduced incentive to compete for non-contracted customers. This might lead contracts to have an "exclusionary foreclosure effect" (Segal and Whinston (2000), Jacobson (2002)).[39] On the other hand, if a long-run contract leads one firm to learn more quickly, which we have seen may be efficient in some circumstances, this may create benefits for non-contracted customers.

We extend our model as follows, and examine outcomes using our $M = 30$ example parameters. Further details and results are in Appendix E.6. There is one repeat and forward-looking buyer, R. Every period, nature chooses R to be the buyer with probability $\theta \in [0, 1)$. Otherwise, the buyer is a myopic/atomistic buyer (A), as assumed previously. Sellers know the buyer's type. Competition for an A buyer is the same as our baseline model, with bargaining weight parameter $\tau^A = 0$, 0.25 or 0.5. We consider three scenarios for how

[39] Our set-up allows us to address a central question in all models of exclusion. For example, Bernheim and Heeb (2014) suggest "[all] exclusionary practices ... share an ability to activate a single economic mechanism. Briefly, the mechanism has the feature that the exclusion from a market or some portion thereof weakens a rival, and thereby impairs its ability to compete for *other* business." [emphasis in the original]

R's purchase is determined, with richer contracts, and an analysis of which scenario is more likely, left for future work.

No contracts. Competition for R's purchase happens period-by-period, as in our standard model, although R is forward-looking and internalizes how its choice will affect its future surplus. We will assume $\tau^R = 0.5$, although the results reported in Appendix Figure E.8 include outcomes when $\tau^R = 0$ and $\tau^A = 0$ as a price-setting comparison.

R has symmetric fixed price contracts with both sellers. R signs identical fixed price contracts with both sellers before the start of the game, and subsequently purchases from each of them with probability 0.5.[40]

R has an exclusive fixed price contract with one seller. R signs an exclusive fixed price contract with one seller before the start of the game, and it always buys from that seller. θ is therefore the proportion of sales that are not available to the firm without the contract. In this scenario one might view θ as what is sometimes called the "foreclosure share". Francis and Sprigman (2023), p. 301, and Hovenkamp (2021) suggest that US case law requires this share to be above 40% for substantial foreclosure to be regarded as likely, although we have not seen such a threshold justified by a formal analysis.

We calculate $HHI^{32,A}$ (expected concentration when there is an A buyer in the 32nd period of the game), $TS^{PDV,A}$ and $CS^{PDV,A}$. In order to avoid the last two measures being mechanically reduced as θ increases, we calculate them as if there is an A buyer in the market every period, although using θ-equilibrium strategies and λ weights. We find equilibria using θ-homotopies for the distinct values of the bargaining parameters that we are considering. There are multiple equilibria if $\tau^R = 0.5$ and $\tau^A = 0$ for low θs. For these parameters, our discussion only considers outcomes on the homotopy paths that go all of the way from $\theta = 0$ to $\theta = 1$. Appendix Figure E.8 presents the results in diagrams, which we summarize here.

The clearest result is that, when we assume $\tau^R = 0.5$, the welfare of A buyers ($CS^{PDV,A}$) is maximized with symmetric contracts, for all $\theta > 0$, and no contracts give higher $CS^{PDV,A}$ than exclusive contracts. Therefore, if one was determining anticompetitive effects using

[40]The fixed price assumption is non-trivial, as dynamic incentives would change with state-contingent prices. This comment is also relevant for an exclusive contract.

Table 1: Ranking of Exclusive Contracts vs. Symmetric Contracts vs. No Contracts in Terms of the Efficiency of Supply to Atomistic Buyers (A) as Functions of θ for $\tau^A = 0$, 0.25 or 0.5. X $\succ$ Y = scenario X has higher $TS^{PDV,A}$ than scenario Y.

	Exclusive Contract $\succ$ Symmetric Contracts	No Contracts $\succ$ Symmetric Contracts	No Contracts $\succ$ Exclusive Contract
$\tau^A = 0$	$0.1 \leq \theta \leq 0.57$	$\theta < 0.69$	$\theta < 0.24$, $\theta > 0.4$
$\tau^A = 0.25$	$0.12 \leq \theta \leq 0.95$	All θ	All θ
$\tau^A = 0.5$	$\theta > 0.26$	$\theta < 0.04$, $\theta > 0.13$	All θ

only the buyer surplus of non-contracted buyers in this model, then these effects would exist below the foreclosure share thresholds suggested by case law.

On the other hand, the patterns for $HHI^{32,A}$ and $TS^{PDV,A}$ are more nuanced and dependent on the parameters. Symmetric contracts minimize expected concentration in the non-contracted markets, but, for each τ^A considered, there is a range of high θs for which no contracts lead to higher expected concentration than exclusive contracts, with the level of concentration depending on the assumed value of τ^A.[41] One intuition for why an exclusive contract could lower concentration in the non-contracted market is that it will make the future sales of the contracted seller less sensitive to whether it sells to non-contracted customers.[42]

Table 1 shows that the relative efficiency of sales to non-contracted customers, measured by $TS^{PDV,A}$, under the three scenarios depends on τ^A, as well as on θ. For example, no contracts are more efficient than symmetric contracts for all θ if $\tau^A = 0.25$, but only limited ranges if $\tau^A = 0$ or $\tau^A = 0.5$. On the other hand, if we assume $\tau^A = 0$, an exclusive contract is more efficient than no contracts if the repeat buyer accounts for between 24 and 40 percent of sales, whereas the exclusive contract is never more efficient than no contracts if $\tau^A = 0.25$ or $\tau^A = 0.5$.

While we should not over-interpret the results of such a stylized model, they do have

[41] For example, if $\tau^A = 0$, no contracts lead to higher concentration, just under 0.8, for $0.8 < \theta < 0.96$, whereas for $\tau^A = 0.5$, no contracts lead to higher concentration, just over 0.99, for $0.3 < \theta < 0.9$.

[42] Consistent with this logic, AB^{PDV} equals around 32 under symmetric contracts around 21.7 for exclusive contracts when $\theta = 0.8$ for both of these τ^As.

interesting implications. For instance, one can imagine an analyst suggesting that, if non-contracted customers have significant bargaining power, long-term exclusive contracts should not be problematic. When considering either the welfare of non-contracted customers or the efficiency of sales to these customers, this claim would fail, at least directionally, in our example.

6 Conclusion

We have investigated whether equilibrium outcomes in a dynamic model with learning-by-doing are sensitive to changes in the standard assumption that prices are set unilaterally by sellers. We find that several important outcomes, such as equilibrium concentration and efficiency, as well as the existence of multiple equilibria, can change significantly when we allow buyers to have even quite modest bargaining power. This happens, in part, because changing bargaining weights not only changes seller markups, but also which sellers are likely to make sales, their expectations of future market structure and dynamic incentives. This is true when we assume seller duopoly and atomistic buyers, in line with the literature, and when we relax these assumptions.

We show how assumptions about bargaining power may also matter for stylized policy applications of dynamic models. For example, assuming seller price-setting, rather than accounting for buyer-seller bargaining, may lead to misleading conclusions about the structure of optimal subsidies, the profitability and welfare implications of horizontal mergers, the possible efficiency benefits to introducing policies that promote more symmetric market structures once a market leader has achieved a sufficient level of know-how, and the efficiency effects of a repeat buyer having long-term contracts. While we understand that our policy examples are very stylized, the results imply that future applications of models with LBD would likely benefit from accounting for how market structure, dynamic incentives and bargaining power interact.

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ONLINE APPENDICES FOR "Dynamic Competition with Bargaining: Implications for Subsidy and Competition Policies" by Deng, Jia, Leccese and Sweeting

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A Analytical Proofs

A.1 Proposition 1.

Proposition 1 *Consider a one-shot bargaining game with seller costs c_1 and c_2,*

1. *for any $\tau \in [0, 1]$, equilibrium prices are unique (i.e., unique (p_1^*, p_2^*) will satisfy equations (4) and (7), or equivalently (8) and (9)).*

2. *the equilibrium markups, $p_i^* - c_i$, and therefore prices, of both firms, strictly decrease in τ. If $c_i > c_j$, then the markup of firm j decreases more than the markup of i.*

3. *if $c_i > c_j$, then $D_i^* < \frac{1}{2}$ and D_i^* is strictly decreasing in τ, and D_j^* is strictly increasing in τ. If $c_i = c_j$, then $D_i^* = \frac{1}{2}$ for all τ.*

4. *expected buyer surplus monotonically increases in τ, and expected producer surplus monotonically decreases in τ. If $c_i \neq c_j$ then expected total surplus monotonically increases in τ.*

Proof of Part 1. We work with our reformulated equilibrium equations. Combining equations (8) and (9), an equilibrium must satisfy

$$\sigma \log \left(\frac{1}{D_1^*} - 1 \right) = c_1 - c_2 + \Phi(D_1^*) - \Phi(1 - D_1^*). \tag{21}$$

The left-hand side (LHS) of (21) is strictly decreasing in D_1^*. We next show that the right-hand side is monotonically increasing.

Note that for $D \in (0,1)$, the markup function $\Phi(D)$ can be rewritten as

$$\Phi(D) = \frac{\sigma}{\frac{\tau}{1-\tau}\frac{D}{\log\frac{1}{1-D}} + 1 - D}. \tag{22}$$

It is straightforward to verify that $\frac{D}{\log\frac{1}{1-D}}$ decreases with $D \in (0,1)$, so $\Phi(D)$ increases. Since $\Phi(D_1^*)$ is increasing in D_1^*, the right-hand side of (21) is increasing.

Combining the monotonicity (in different directions) of the two sides of (21) with respect to D_1^*, and the properties that $\lim_{D_1^* \to 0} LHS(D_1^*) = +\infty$ and $\lim_{D_1^* \to 1} LHS(D_1^*) = -\infty$ completes the proof.

Proof of Part 2. First, it is useful to note that for any fixed $D \in (0,1)$, the markup $\Phi(D;\tau)$ is strictly decreasing in $\tau \in [0,1]$, which can be clearly seen by rewriting $\Phi(D;\tau)$ in the following way:

$$\Phi(D;\tau) = \frac{\sigma \log\frac{1}{1-D}}{\frac{\tau}{1-\tau}D + (1-D)\log\frac{1}{1-D}}. \tag{23}$$

Then, if $c_1 = c_2$, we know that $D_1^* = D_2^* = \frac{1}{2}$ in equilibrium, so the equilibrium markup $\Phi(\frac{1}{2})$ is strictly decreasing in $\tau \in [0,1]$. It remains to check the $c_1 \neq c_2$ case.

Suppose that $c_1 > c_2$ without loss of generality. As τ increases, we know from Part 3 (proved below) that $D_1^*(\tau)$ decreases. From the proof of Part 1, we know that $\Phi(D;\tau)$ is increasing in D. As a result, the changes in both τ and in $D_1^*(\tau)$ cause the markup $\Phi(D_1^*(\tau);\tau)$ to strictly decrease.

Recall the equilibrium condition $\sigma \log\left(\frac{1}{D_1^*(\tau)} - 1\right) = c_1 - c_2 + \Phi(D_1^*;\tau) - \Phi(D_2^*;\tau)$, which can be rewritten as

$$\Phi(D_2^*(\tau);\tau) = c_1 - c_2 + \Phi(D_1^*(\tau);\tau) - \sigma \log\left(\frac{1}{D_1^*(\tau)} - 1\right). \tag{24}$$

Since $\Phi(D_1^*(\tau);\tau)$ strictly decreases with τ and $\log\left(\frac{1}{D_1^*(\tau)} - 1\right)$ strictly increases with τ, we have that $\Phi(D_2^*(\tau);\tau)$ strictly decreases with τ. Moreover, it decreases more than $\Phi(D_1^*(\tau);\tau)$ does.

Proof of Part 3. For the $c_i > c_j$ part, suppose that $c_1 > c_2$ without loss of generality. This part is implied by the proof of Proposition 3 below. Specifically if $\beta = 0$, then the equation for the state (2,1) is the same as the equation for any state where $c_1 = c(1)$ and $c_2 = c(2)$. That is, with $\beta = 0$, $c(1) = c_1$, and $c(2) = c_2$, (25) is equivalent to (21). So the analysis of (25) which establishes that $D_1^*(1,2) < \frac{1}{2}$ and that $D_1^*(1,2)$ is strictly decreasing in τ also applies to (21).

If $c_i = c_j$, then it follows from equations (8) and (9) that the only equilibrium is symmetric with $D_1^* = D_2^* = \frac{1}{2}$.

Proof of Part 4. We first consider the monotonicity of expected total surplus with respect to $\tau \in (0,1)$. Expected total surplus can be expressed in terms of market shares as follows.

$$TS(D_1^*, D_2^*) = D_1^* \left[\sigma \log\left(\frac{1}{D_1^*}\right) - c_1\right] + D_2^* \left[\sigma \log\left(\frac{1}{D_2^*}\right) - c_2\right].$$

Suppose that $c_1 > c_2$ without loss of generality. Since $D_2^* = 1 - D_1^*$, the total derivative of TS with respect to D_1^* is

$$\sigma \log\left(\frac{1}{D_1^*} - 1\right) - (c_1 - c_2) = \Phi(D_1^*) - \Phi(1 - D_1^*) < 0,$$

where the equality follows from the equilibrium condition (21), and the inequality is because $D_1^* < D_2^*$ by Part 3, and $\Phi(D)$ strictly increases with D. This implies that TS is strictly decreasing in D_1^*. By Part 3, D_1^* strictly decreases with τ. As a result, TS increases with τ monotonically. At $\tau = 1$, the sellers' markup is 0 and thus the social planner outcome is achieved.

Next, we show that the expected producer surplus is decreasing in $\tau \in (0,1)$. The expected producer surplus is

$$PS(p_i^*, p_j^*) = \sum_{i=1,2} (p_i^* - c_i) \frac{\exp\left(-\frac{p_i^*}{\sigma}\right)}{\sum_{j=1,2} \exp\left(-\frac{p_j^*}{\sigma}\right)}.$$

We have that

$$\frac{\partial PS(p_i^*, p_j^*)}{\partial p_i^*} = \frac{D_i^*}{\sigma}\left[\sigma - (1 - D_i^*)(p_i^* - c_i) + (1 - D_i^*)(p_j^* - c_j)\right].$$

Recall that the bargaining FOC for seller i is

$$-\tau D_i^*(p_i^* - c_i) + (1 - \tau)\left[\sigma - (1 - D_i^*)(p_i^* - c_i)\right] \log \frac{1}{1 - D_i^*} = 0,$$

which implies that $\sigma - (1 - D_i^*)(p_i^* - c_i) = \frac{\tau D_i^*(p_i^* - c_i)}{(1-\tau)\log\frac{1}{1-D_i^*}} > 0$. Therefore, $\frac{\partial PS(p_i^*, p_j^*)}{\partial p_i^*} > 0$. Since we have shown in Part 3 that markups, and thus prices, decrease with τ, it is clear that PS decreases with τ.

Finally, it is straightforward to conclude that buyer surplus increases with τ because it is equal to $TS - PS$.

A.2 Proposition 2.

Proposition 2 *In a model with any $m \leq M$, there will be a unique symmetric MPNE if*

1. *$\delta = 0$, for any ρ and τ, or*
2. *$\tau = 1$, for any ρ and δ.*

Proof of Part 1. If $\delta = 0$ then, once the game leaves a state it cannot return, and the game must eventually end up in state (M, M) and stay there forever. One can then apply the recursive proof of Besanko, Doraszelski, Kryukov, and Satterthwaite (2010), Proposition 3, using the fact that our Proposition 1 implies that, for any τ, there is a unique pricing equilibrium given opportunity costs.[43]

Proof of Part 2. If $\tau = 1$, then from equation (9), markups, $\Phi(D_1^*(\mathbf{e}))$, are zero for all states, and from (12) prices equal production costs, i.e., $p_1^*(\mathbf{e}) = c_1(\mathbf{e})$, in all states.

[43] We assume that $\delta < 1$, but a proof of uniqueness if $\delta = 1$ follows the same logic.

A.3 Proof of Proposition 3.

Proposition 3 *For $m = M = 2$ and $\delta = 0$, the unique symmetric equilibrium will have the following properties*

1. *equilibrium $D_1^*(1,2)(\tau) < \frac{1}{2}$ for all τ.*
2. *equilibrium $D_1^*(1,2)(\tau)$ is strictly decreasing in τ.*
3. *for $t \geq 2$, HHI^t is strictly increasing in τ.*
4. *there exists a τ^* such that $TS^{PDV}(\tau^*) = TS^{SP}$, $TS^{PDV}(\tau)$ is strictly increasing in τ for $\tau \in (0, \tau^*)$ and strictly decreasing in τ for $\tau \in (\tau^*, 1)$.*

A.3.1 Characterization

Given $M = 2$ and $\delta = 0$, the equilibrium is fully characterized by the equilibrium condition for $D_1^*(1,2)$, which can be simplified to

$$H(x,\tau) - H\left(\frac{1}{2},\tau\right) = c(1) - c(2), \tag{25}$$

where $H(x)$ is defined as

$$H(x,\tau) := \log\frac{1-x}{x} - \frac{1-\beta+\beta x}{1-\beta}\left[\widetilde{\Phi}\left(x, \frac{\tau}{1-\tau}\right) - (1-\beta)\widetilde{\Phi}\left(1-x, \frac{\tau}{1-\tau}\right)\right] \text{ for } x,\tau \in [0,1], \tag{26}$$

where $\widetilde{\Phi}(x,z)$ is defined as

$$\widetilde{\Phi}(x,z) := \frac{\log\frac{1}{1-x}}{zx + (1-x)\log\frac{1}{1-x}} \text{ for } x \in [0,1] \text{ and } z \in [0,\infty),^{44} \tag{27}$$

and is a reformulated version of the mark-up condition $\Phi(x,\tau)$ where the second argument in $\widetilde{\Phi}(x,z)$ is $\frac{\tau}{1-\tau}$.

As negotiated prices are a transfer from buyers to sellers, we can express expected total surplus in any state as a function of the choice probabilities and firm costs only, i.e.,

[44] $\widetilde{\Phi}(x,\infty)$ is defined to be 0 for all $x \in [0,1]$.

$\sum_{k=1,2} D_k(\mathbf{e})\left(\log \frac{1}{D_k(\mathbf{e})} - c_k(\mathbf{e})\right)$ which does not depend on τ. In the $M = 2$ and $\delta = 0$ case, TS^{PDV}, for a game starting at (1,1), can be written as the following function of $D_1(1,2)$,

$$
\begin{aligned}
TS^{PDV}(x) = & \frac{\beta}{1-\beta+\beta x}\left[x \log \frac{1}{x} + (1-x)\log \frac{1}{1-x} - (c(1)-c(2))x - \log 2\right] + \\
& \frac{\log 2 - \beta c(2) - (1-\beta)c(1)}{1-\beta}.
\end{aligned}
$$

We define D_1^{SP} as the choice probability that the social planner would choose in state (1,2). The social planner, would, of course, use choice probabilities of $\frac{1}{2}$ in states (1,1) and (2,2) where the suppliers are symmetric, with the firm with the highest ε making the sale.

When $\delta = 0$ and $x = D_1(1,2)$,

$$
HHI^t = (x^2 + (1-x)^2)(1-x)^{t-1} + \frac{1}{2}(1-(1-x)^{t-1}) \tag{28}
$$

for $t \geq 2$ as the state will either be (1,2) with concentration $(x^2 + (1-x)^2)$ or (2,2) with concentration $\frac{1}{2}$.

A.3.2 Preliminary Results.

Lemma A.1 *$H(x,\tau)$ is strictly decreasing in $x \in (0,1)$.*

Proof of Lemma A.1. Since there is a unique equilibrium when $\delta = 0$ for any parameterization of $c(e)$, $H(x,\tau)$ must be strictly monotone in $x \in (0,1)$. Otherwise, there will be multiple equilibria—i.e., multiple solutions to (25)—for some value of $c(1) - c(2)$.

It suffices to show that $H(x,\tau)$ is decreasing on some interval in $(0,1)$. When $\tau = 1$, it is easy to see that $H(x,\tau)$ is strictly decreasing $x \in (0,1)$. When $\tau < 1$, we have that $\widetilde{\Phi}\left(x, \frac{\tau}{1-\tau}\right)$ strictly increases with $x \in (0,1)$. Therefore, $H(x,\tau)$ decreases with x when $\widetilde{\Phi}\left(x, \frac{\tau}{1-\tau}\right) > (1-\beta)\widetilde{\Phi}\left(1-x, \frac{\tau}{1-\tau}\right)$. This completes the proof. ∎

Lemma A.2 *$TS^{PDV}(x)$ is strictly increasing in $x \in (0, D_1^{SP})$ and strictly decreasing in $x \in (D_1^{SP}, 1)$, where $D_1^{SP} \in (0, 1/2)$ solves*

$$
\log \frac{1}{x} - \frac{1}{1-\beta}\log \frac{1}{1-x} = c(1) - c(2) - \frac{\beta}{1-\beta}\log 2. \tag{29}
$$

Proof of Lemma A.2. The proof immediately follows from the fact that

$$\frac{dTS^{PDV}(x)}{dx} = \frac{\beta(1-\beta)}{(1-\beta+\beta x)^2}\left\{\log\frac{1}{x} - \frac{1}{1-\beta}\log\frac{1}{1-x} - \left[c(1) - c(2) - \frac{\beta}{1-\beta}\log 2\right]\right\}. \tag{30}$$

■

A.3.3 Proofs of Proposition 3

Proof of Proposition 3.1: equilibrium $D_1^*(1,2)(\tau) < \frac{1}{2}$ for all τ. By Lemma A.1, the left-hand side of (25) is strictly decreasing in x. It is clear that the left-hand side equals 0 when $x = \frac{1}{2}$. Since the right-hand side is strictly positive, the solution must be less than $\frac{1}{2}$.

■

Proof of Proposition 3.2: equilibrium $D_1^*(1,2)(\tau)$ is strictly decreasing in τ. Applying the implicit function theorem to (25) yields that

$$\frac{\partial x}{\partial \tau} = -\frac{H_\tau(x,\tau) - H_\tau\left(\frac{1}{2},\tau\right)}{H_x(x,\tau)}. \tag{31}$$

By Lemma A.1, $H_x(x,\tau) < 0$, and by Proposition 3.1, $x < \frac{1}{2}$. So to complete the proof, it suffices to show that $H_\tau(x,\tau) < H_\tau\left(\frac{1}{2},\tau\right)$ for $x \in \left(0,\frac{1}{2}\right)$.

Note that

$$H_\tau(x,\tau) = -\frac{1-\beta+\beta x}{1-\beta}\left[\widetilde{\Phi}_z(x,z) - (1-\beta)\widetilde{\Phi}_z(1-x,z)\right]\frac{dz}{d\tau}, \text{ with } z = \frac{\tau}{1-\tau}. \tag{32}$$

Therefore,

$$\begin{aligned}
&H_\tau(x,\tau) < H_\tau\left(\frac{1}{2},\tau\right) \\
\iff & -\frac{1-\beta+\beta x}{1-\beta}\left[\widetilde{\Phi}_z(x,z) - (1-\beta)\widetilde{\Phi}_z(1-x,z)\right] < -\frac{(2-\beta)\beta}{2(1-\beta)}\widetilde{\Phi}_z\left(\frac{1}{2},z\right) \\
\iff & -\left[\widetilde{\Phi}_z(x,z) - (1-\beta)\widetilde{\Phi}_z(1-x,z)\right] < -\frac{(1-\beta)(2-\beta)\beta}{2(1-\beta)(1-\beta+\beta x)}\widetilde{\Phi}_z\left(\frac{1}{2},z\right).
\end{aligned} \tag{33}$$

In fact,

$$\widetilde{\Phi}_z(x,z) = -\frac{x\log\frac{1}{1-x}}{\left[zx+(1-x)\log\frac{1}{1-x}\right]^2}, \text{ and } \widetilde{\Phi}_z\left(\frac{1}{2},z\right) = -\frac{2\log 2}{(z+\log 2)^2} < 0. \tag{34}$$

So the right-hand side of (33) is decreasing in $x \in \left(0, \frac{1}{2}\right)$.

Next, we show that the left-hand side of (33) increases with $x \in \left(0, \frac{1}{2}\right)$. It suffices to show that $-\widetilde{\Phi}_z\,(x,z)$ increases with $x \in \left(0, \frac{1}{2}\right)$. In fact,

$$\begin{aligned} -\widetilde{\Phi}_{zx}\,(x,z) &= \frac{\left(\log\frac{1}{1-x}+\frac{x}{1-x}\right)\left[zx+(1-x)\log\frac{1}{1-x}\right]-2x\log\frac{1}{1-x}\left(z+1-\log\frac{1}{1-x}\right)}{\left[zx+(1-x)\log\frac{1}{1-x}\right]^3} \\ &= \frac{zx\left(\frac{x}{1-x}-\log\frac{1}{1-x}\right)+\log\frac{1}{1-x}\left[(1+x)\log\frac{1}{1-x}-x\right]}{\left[zx+(1-x)\log\frac{1}{1-x}\right]^3}. \end{aligned}$$

It is easy to verify that $\frac{x}{1-x} > \log\frac{1}{1-x} > \frac{x}{1+x}$ for $x \in \left(0, \frac{1}{2}\right)$. As a result, $-\widetilde{\Phi}_{zx}\,(x,z) > 0$.

Combining the facts that the right-hand side of (33) is decreasing in $x \in \left(0, \frac{1}{2}\right)$, the left-hand side of (33) is increasing in $x \in \left(0, \frac{1}{2}\right)$, and both sides are equal at $x = \frac{1}{2}$, we can conclude that (33) holds for $x \in \left(0, \frac{1}{2}\right)$. This completes the proof. ■

Proof of Proposition 3.3: equilibrium expected concentration (HHI^t) in any period $t \geq 2$, strictly increases in τ. The derivative of HHI^t with respect to x is, for $t \geq 2$

$$-\frac{(1-2x)(1-x)^{t-2}(3+t-2x-2tx)}{2}$$

which is negative for any $x < \frac{1}{2}$. As equilibrium $D_1^*(1,2) < \frac{1}{2}$ (Proposition 3.1) and equilibrium $D_1^*(1,2)$ decreases in τ (Proposition 3.2), HHI^t increases in τ. ■

Proof of Proposition 3.4 : there exists a τ^* such that $TS^{PDV}(\tau^*) = TS^{SP}$, and $TS^{PDV}(\tau^*)$ is increasing in τ for $\tau \in (0,\tau^*)$ and decreasing in τ for $\tau \in (\tau^*,1)$.

As TS^{PDV} only depends on $D_1^*(1,2)$, $TS^{PDV}(\tau^*) = TS^{SP}$ if $D_1^*(1,2)(\tau^*) = D_1^{SP}$. Because $D_1^*(1,2)(\tau)$ decreases with τ (Proposition 3.2), and Lemma A.2, the result follows if

$$D_1^*(1,2)(\tau=0) > D_1^{SP} > D_1^*(1,2)(\tau=1).$$

By (25) and the monotonicity of $H(x,\tau)$ in x (Lemma A.1), the above inequality is equivalent to

$$H(D_1^{SP},0) - H\left(\frac{1}{2},0\right) - [c(1)-c(2)] > 0 > H(D_1^{SP},1) - H\left(\frac{1}{2},1\right) - [c(1)-c(2)]. \quad (35)$$

We first show that $0 > H(D_1^{SP},1) - H\left(\frac{1}{2},1\right) - [c(1)-c(2)]$. In fact,

$$H(D_1^{SP},1) - H\left(\frac{1}{2},1\right) - [c(1)-c(2)] = \frac{\beta}{1-\beta}\left(\log\frac{1}{1-D_1^{SP}} - \log 2\right) < 0,$$

where the equality is due to that D_1^{SP} solves (29), and the inequality is due to that $D_1^{SP} < \frac{1}{2}$ (Lemma A.2).

It only remains to show that $H(D_1^{SP},0) - H\left(\frac{1}{2},0\right) - [c(1)-c(2)] > 0$. Note that

$$\begin{aligned}
&H(D_1^{SP},0) - H\left(\frac{1}{2},0\right) - [c(1)-c(2)] \\
&= \log\left(\frac{1-D_1^{SP}}{D_1^{SP}}\right) - [c(1)-c(2)] - \left[\frac{1}{(1-\beta)(1-D_1^{SP})} - \frac{1-\beta}{D_1^{SP}} - \frac{\beta(2-\beta)}{1-\beta}\right] - H\left(\frac{1}{2},0\right).
\end{aligned}$$

Again, substituting (29) into the right-hand side of the above equation yields that

$$H(D_1^{SP},0) - H\left(\frac{1}{2},0\right) - [c(1)-c(2)] = L(D_1^{SP}) - L\left(\frac{1}{2}\right),$$

where

$$L(x) := \frac{\beta}{1-\beta}\log\frac{1}{1-x} - \left[\frac{1}{(1-\beta)(1-x)} - \frac{1-\beta}{x}\right].$$

Since

$$L'(x) = -\frac{1-\beta(1-x)}{(1-\beta)(1-x)^2} - \frac{1-\beta}{x^2} < 0,$$

$L(x)$ is decreasing in x, and $L(D_1^{SP}) > L\left(\frac{1}{2}\right)$. Therefore

$$H(D_1^{SP},0) - H\left(\frac{1}{2},0\right) - [c(1)-c(2)] = L(D_1^{SP}) - L\left(\frac{1}{2}\right) > 0.$$

■

B Numerical Methods for Finding Equilibria

B.1 Homotopies.

This Appendix provides details of our implementation of the homotopy algorithm using the example of how we use a sequence of homotopies to try to enumerate the number of equilibria that exist for different values of (ρ, δ) for given values of τ. Our implementation of other homotopies, for example, by varying τ, is similar to a single step in this sequence. We describe the procedure assuming that $M = 30$ and that we are using the price and value formulation of the equations. Identical procedures apply using the choice probability formulation, except that it is necessary to use the choice probabilities to calculate prices and values in order to check whether the identified solutions are different enough to be labeled as distinct equilibria.

B.1.1 Preliminaries

We identify equilibria at particular gridpoints in (ρ, δ) space. We specify a 201-point evenly-spaced grid for the forgetting rate $\delta \in [0, 0.2]$ and a 41-point evenly-spaced grid for the learning progress ratio $\rho \in [0.6, 1]$. The state space of the game is defined by a (30×30) grid of values of the know-how of each firm.

B.1.2 System of Equations Defining Equilibrium

An MPNE is defined by a system of equations (one V^{S*} equation (text equation (2)) for each of 900 states and one p^* equation (text equation (6)) for each of 900 states. The grouping of all of these equations is denoted F.

B.1.3 Homotopy Algorithm: Overview

The idea of the homotopy is to trace out an equilibrium correspondence as one of the parameters of interest is changed, holding the others fixed. Starting from any equilibrium, the numerical algorithm traces a path where a parameter (such as δ), and the vectors $V^{S*}(\mathbf{e})$ and $p^*(\mathbf{e})$ are changed together so that the equations F continue to hold, by solving a system of differential equations. The differential equation solver does not return equilibria exactly at the gridpoints so it is necessary to interpolate between the solutions returned by the solver. Homotopies can be run starting from different equilibria and varying different parameters. When these different homotopies return interpolated solutions at the same gridpoint it is necessary to define a numerical rule for when two different solutions should be counted as different equilibria.

B.1.4 Procedure Details

Step 1: Finding Equilibria for $\delta = 0$. The first step is to find an equilibrium (i.e., a solution to the 1,800 equations) for $\delta = 0$ for each value of ρ on the grid. There will be a unique MPNE for $\delta = 0$, as, in this case, movements through the state space are unidirectional, so that the state will eventually end up in the state (M, M) where no more learning is possible.

We solve for an equilibrium using the Levenberg-Marquardt algorithm implemented using `fsolve` in MATLAB, where we supply analytic gradients for each equation. The solution for the previous value of ρ are used as starting values. To ensure that the solutions are precise, we use a tolerance of 10^{-7} for the sum of squared values of each equation, and a relative tolerance of 10^{-14} for the price and value variables that we are solving for.

Step 2: δ-Homotopies. Using the notation of Besanko, Doraszelski, Kryukov, and Satterthwaite (2010), we explore the correspondence

$$F^{-1}(\rho) = \{(\mathbf{V}^*, \mathbf{p}^*, \delta) | F(\mathbf{V}^*, \mathbf{p}^*; \rho, \delta) = \mathbf{0}, \quad \delta \in [0, 1]\},$$

The homotopy approach follows the correspondence as a parameter, s, changes (in our analysis, s could be δ, ρ or τ). Denoting $\mathbf{x} = (\mathbf{V}^*, \mathbf{p}^*)$, $F(\mathbf{x}(s), \delta(s), \rho) = \mathbf{0}$ can be implicitly differentiated to find how $\mathbf{x}$ and δ must change for the equations to continue to hold as s changes.

$$\frac{\partial F(\mathbf{x}(s), \delta(s), \rho)}{\partial \mathbf{x}} \mathbf{x}'(s) + \frac{\partial F(\mathbf{x}(s), \delta(s), \rho)}{\partial \delta} \delta'(s) = \mathbf{0}$$

where $\frac{\partial F(\mathbf{x}(s), \delta(s), \rho)}{\partial \mathbf{x}}$ is a (1,800 x 1,800) matrix, $\mathbf{x}'(s)$ and $\frac{\partial F(\mathbf{x}(s), \delta(s), \rho)}{\partial \delta}$ are both (1,800 x 1) vectors and $\delta'(s)$ is a scalar. The solution to these differential equations will have the following form, where $y_i'(s)$ is the derivative of the ith element of $\mathbf{y}(s) = (\mathbf{x}(s), \delta(s))$,

$$y_i'(s) = (-1)^{i+1} \det \left(\left(\frac{\partial F(\mathbf{y}(s), \rho)}{\partial \mathbf{y}} \right)_{-i} \right)$$

where $_{-i}$ means that the ith column is removed from the (1,801 x 1,801) matrix $\frac{\partial F(\mathbf{y}(s),\rho)}{\partial \mathbf{y}}$.

To implement the path-following procedure, we use the FORTRAN routine FIXPNS from HOMPACK90, with the ADIFOR 2.0D automatic differentiation package used to evaluate the sparse Jacobian $\frac{\partial F(\mathbf{y}(s),\rho)}{\partial \mathbf{y}}$ and the STEPNS routine is used to find the next point on the path.[45,46]

The FIXPNS routine will return solutions at values of δ that are not equal to the gridpoints. Therefore we adjust the code so that after *each* step, the algorithm checks whether a gridpoint has been passed and, if so, the routine ROOTNX is used to calculate the equilibrium at the gridpoint, using information on the solutions at either side.[47]

The time taken to run a homotopy is usually between one hour and seven hours, when it is run on UMD's BSWIFT cluster (a moderately sized cluster for the School of Behavioral and Social Sciences).

Step 3: Enumerating Equilibria. Once we have collected the solutions at each of the (ρ, δ) gridpoints we need to identify which solutions represent distinct equilibria, taking into account that small differences may arise because of numerical differences that are within our tolerances. For this paper, we use the rule that solutions count as different equilibria if at least one element of the price vector differs by more than 0.001.

[45]STEPNS is a predictor-corrector algorithm where hermetic cubic interpolation is used to guess the next point, and an iterative procedure is then used to return to the path.

[46]For details of the HOMPACK subroutines, please consult manual of the algorithm at `https://users.wpi.edu/~walker/Papers/hompack90,ACM-TOMS_23,1997,514-549.pdf`.

[47]It can happen that the ROOTNX routine stops prematurely so that the returned solution is not exactly at the gridpoint value of δ. We do not use the small proportion of solutions where the difference is more than 10^{-6}. Varying this threshold does not affect the reported results. We also need to decide whether the equations have been solved accurately enough so that the values and strategies can be treated as equilibria. The criteria that we use is that solutions where the value of each equation residual should be less than 10^{-10}. Otherwise, the solution is rejected. In practice, the rejected solutions typically have residuals that are much larger than 10^{-10}.

Step 4: ρ-Homotopies. With a set of equilibria from the δ-homotopies in hand, we can perform the next round of our criss-crossing procedure which alternates ρ-homotopies and δ-homotopies, which we run in both directions (e.g., decreasing ρ as well as increasing ρ). We use equilibria found in the last round as starting points.[48]

This second round of homotopies can also help us to deal with gridpoints where the first round δ-homotopies identify no equilibria because a homotopy run stops (or takes a long sequence of infinitesimally small steps). As noted by Besanko, Doraszelski, Kryukov, and Satterthwaite (2010) (p. 467), the homotopies may stop if they reach a point where the evaluated Jacobian $\frac{\partial F(\mathbf{y}(s),\rho)}{\partial \mathbf{y}}$ has less than full rank. Suppose, for example, that the δ-homotopy for $\rho = 0.8$ stops at $\delta = 0.1$, so we have no equilibria for δ values above 0.1. Homotopies that are run from gridpoints where we did find equilibria with higher values of δ and higher or lower values of ρ may fill in some of the missing equilibria.

Step 5: Repeat Steps 3, 2 and 4 to Identify Additional Equilibria Using New Equilibria as Starting Points. We use the procedures described in Step 3 to identify new equilibria at the gridpoints. These new equilibria are used to start new sets of δ-homotopies, which in turn can identify equilibria that can be used for new sets of ρ-homotopies. This iterative process is continued until the number of additional equilibria that are identified in a round has no noticeable effect on the heatmaps which show the number of equilibria. For the Besanko, Doraszelski, Kryukov, and Satterthwaite (2010), $\tau = 0$ case, this happens after 8 rounds.

B.2 Method for Finding Equilibria Based on Three Reformulated Equations in the $M = 3$ Model.

We now describe the alternative method that we use to identify equilibria when $M = 3$.

As described in the text, the equilibrium conditions can be reformulated in terms of the probability that seller 1 is chosen in each state. If we restrict ourselves to symmetric

[48] In practice, using all new equilibria could be computationally prohibitive. We therefore use an algorithm that continues to add new groups of 10,000 starting points when we find that using additional starting points yields a significant number of equilibria that have not been identified before. We have experimented with different rules, and have found that alternative algorithms do not find noticeably more equilibria, across the parameter space, than the algorithm that we use.

equilibria then, together with the restriction that $D_1(e_1, e_2) = 1 - D_1(e_2, e_1)$, then there are just three unknown probabilities. We will use $D_1(1,2)$, $D_1(1,3)$ and $D_1(2,3)$. The equilibrium equations for these three states are:

$$\sigma \log\left(\frac{1}{D_1^*(e_1, e_2)} - 1\right) - p_1^*(e_1, e_2) + p_2^*(e_1, e_2) = 0, \tag{36}$$

and, from text Section 3.2,

$$\mathbf{p_1} = \mathbf{\Phi}(\mathbf{D_1}) + \mathbf{c_1} - \beta(\mathbf{Q_1} - \mathbf{Q_2})(\mathbf{I} - \beta\mathbf{Q_2})^{-1}[\mathbf{D_1} \circ \mathbf{\Phi}(\mathbf{D_1})]. \tag{37}$$

in vector form, so that we can substitute prices to express the equations (36) in terms of choice probabilities only.

We proceed in the following steps for a given (ρ, δ, τ) combination.

Step 1. Define a grid of possible values for $D_1(1,2)$ and $D_1(1,3)$. For each, we use a vector [1e-10, 1e-9, 1e-7, 1e-6, 1e-5, (0.0001:(0.9999-0.0001)/200:0.9999), 1-1e-5, 1-1e-6, 1-1e-7, 1-1e-8, 1-1e-9, 1-1e-10].

Step 2. For every combination on the grid, find for the value of $D_1(2,3)$ which solves the equilibrium equation for state (2,3), and record the values of the equations (36) for states (1,2) and (1,3), in matrices $C(1,2)$ and $C(1,3)$.[49]

Step 3. Use MATLAB `contour` command to define the shapes where the $C(1,2)$ and $C(1,3)$ surfaces are equal to zero.

Step 4. Count all of the intersections of these curves, using the user-defined MATLAB function `InterX` command.[50]

Of course, the contours are calculated using interpolation so the solutions are therefore not quite exact. Therefore,

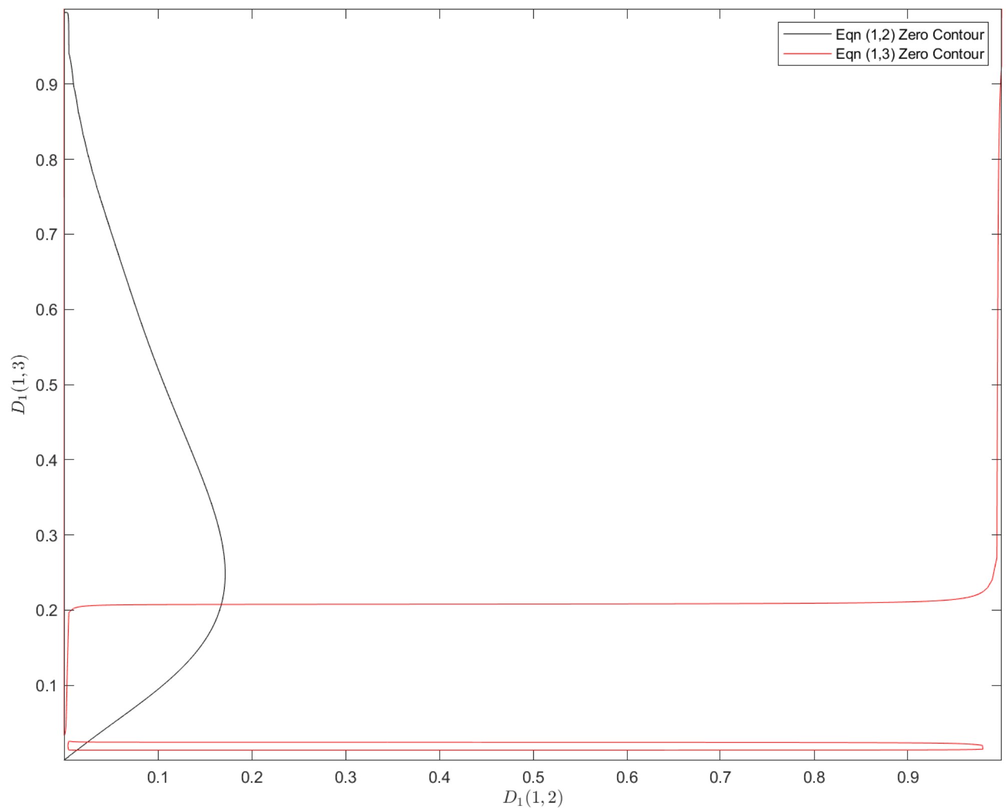
Step 5. Using the solutions from the contour intersections as starting points, solve the equilibrium equations using `fsolve`.

Step 6. Count the number of solutions where at least one choice probability is different

[49] We have not been able to prove uniqueness, but all of the the examples we have looked at there is a unique solution.

[50] `https://www.mathworks.com/matlabcentral/fileexchange/22441-curve-intersections`.

Figure B.2: Illustration of the Contour Plot for $\rho = 0.1$, $\delta = 0.05$ and $\tau = 0$.



from all of the other equilibria by at least 5e-4.

To give a sense of the procedure, consider the parameters $\rho = 0.1$, $\delta = 0.05$ and $\tau = 0.0$. Figure B.2 shows the contour plot. The three intersections between the black and red lines in the bottom left of the figure identify equilibria.

C Additional Analysis for the $M = 3$ Model

This Appendix provides some additional analysis for the $M = 3$ model, including for results that are mentioned briefly in the text.

C.1 Equilibrium Strategies and Outcomes for Polar Cases for $\rho = 0.3$ and $\delta = 0.03$.

We use $\rho = 0.3$ and $\delta = 0.03$ as our leading example parameters for the $M = 3$ model. Table C.2 shows equilibrium prices, sale probabilities and welfare outcomes for (i) the social planner solution, (ii) the equilibrium when $\tau = 0$, and (iii) the equilibrium when $\tau = 1$. The table also shows the probabilities that the industry is in each state after 4 periods (note that state (3,3) cannot be reached in this case) and 32 periods.

C.2 Buyer and Producer Surplus as Functions of τ with Static Seller Behavior and Fixed Production Costs.

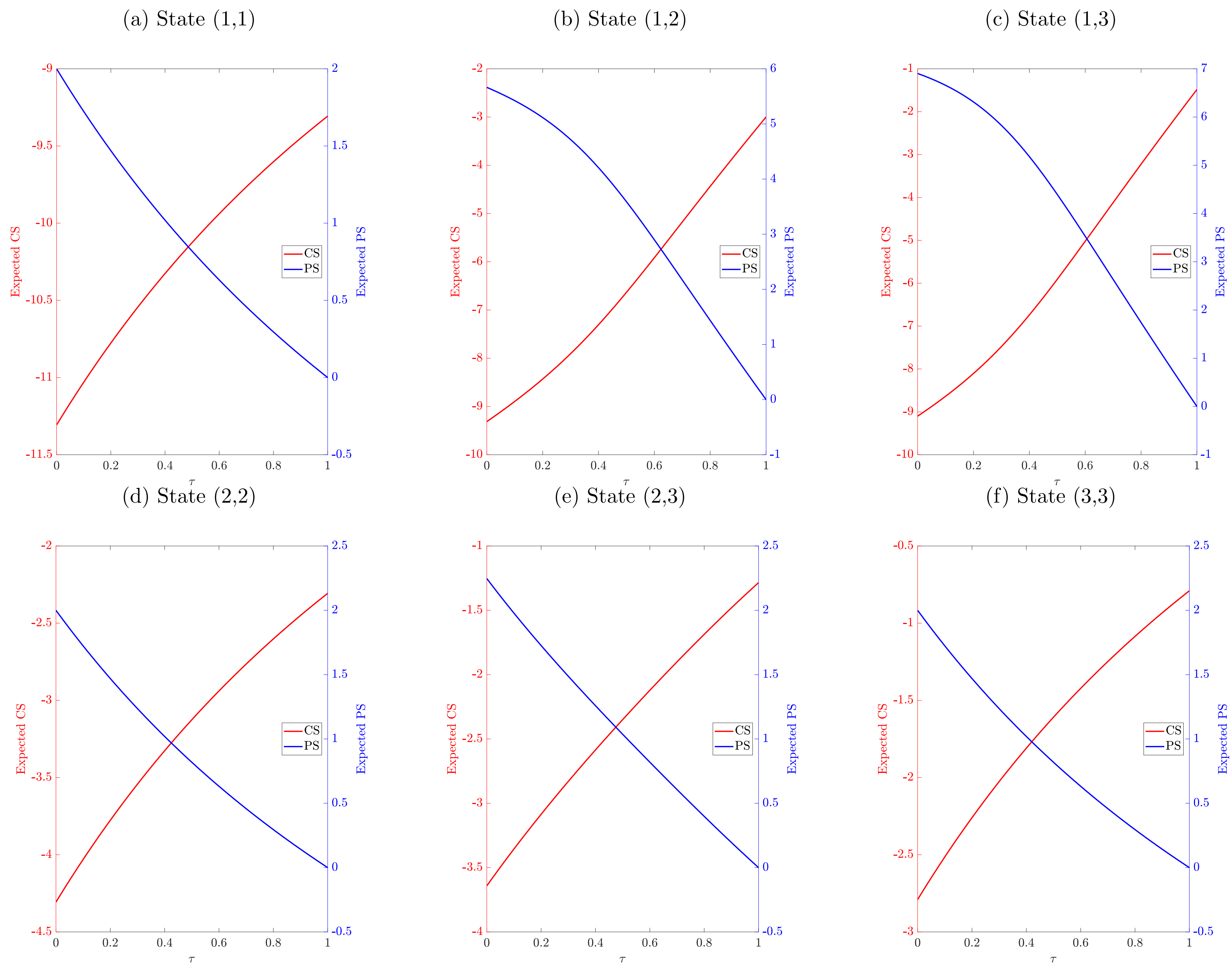
Figure 2(d) in the main text shows that, in the dynamic model, buyer surplus has a non-monotonic relationship with τ, and that both buyer and producer surplus change are much more sensitive to changes in τ when τ is small. Our discussion comments that, in a static, i.e. one-shot, model, the relationships would not only be monotonic, but also often "close to linear". We illustrate this claim by looking at how expected surplus changes if we assume that sellers are static with fixed production costs.[51] Figure C.2 shows the results for the six states in the model (recognizing that for this purpose (e_1, e_2) and (e_2, e_1) are the same state when calculating expected surplus). In each case, both the buyer surplus and producer surplus lines curve only gently as τ varies.

[51] One can consider many other values of costs. The only differences that arise is that, when choice probabilities are very close to one an increase in τ has a smaller marginal effect on the surplus measures.

Table C.2: $M = 3$ with $\rho = 0.3$ and $\delta = 0.03$: Strategies and Outcomes in Polar Cases. In these tables, firm 2 is assumed to be the leader.

	(a) Social Planner			**(b) $\tau = 0$ Equilibrium**			**(c) $\tau = 1$ Equilibrium**		
	Laggard State Firm 1			Laggard State Firm 1			Laggard State Firm 1		
e_1	1	2	3	1	2	3	1	2	3
c_1	10	3	1.483	10	3	1.483	10	3	1.483
Δ	0.03	0.059	0.087	0.03	0.059	0.087	0.03	0.059	0.087
Prices and Laggard Sale Probability									
$e_2 = 1$	$p_1 = 10$ $p_2 = 10$ $D_1 = 0.5$			$p_1 = 3.378$ $p_2 = 3.378$ $D_1 = 0.5$			$p_1 = 10$ $p_2 = 10$ $D_1 = 0.5$		
$e_2 = 2$	$p_1 = 10$ $p_2 = 3$ $D_1 = 0.084$	$p_1 = 3$ $p_2 = 3$ $D_1 = 0.5$		$p_1 = 4.355$ $p_2 = 3.274$ $D_1 = 0.253$	$p_1 = 2.918$ $p_2 = 2.918$ $D_1 = 0.500$		$p_1 = 10$ $p_2 = 3$ $D_1 = 0.001$	$p_1 = 3$ $p_2 = 3$ $D_1 = 0.500$	
$e_2 = 3$	$p_1 = 10$ $p_2 = 1.483$ $D_1 = 0.114$	$p_1 = 3$ $p_2 = 1.483$ $D_1 = 0.58$	$p_1 = 1.483$ $p_2 = 1.483$ $D_1 = 0.5$	$p_1 = 4.344$ $p_2 = 3.395$ $D_1 = 0.279$	$p_1 = 3.059$ $p_2 = 3.225$ $D_1 = 0.542$	$p_1 = 3.339$ $p_2 = 3.339$ $D_1 = 0.5$	$p_1 = 10$ $p_2 = 1.483$ $D_1 = 0.000$	$p_1 = 3$ $p_2 = 1.483$ $D_1 = 0.180$	$p_1 = 1.483$ $p_2 = 1.483$ $D_1 = 0.5$
4 Period Probability Distribution									
$e_2 = 1$	4.57E-05			8.18E-05			2.72E-05		
$e_2 = 2$	0.0070	0.0190		0.0087	0.050		0.0060	0.0001	
$e_2 = 3$	0.8233	0.1506	0	0.5741	0.3670	0	0.9930	0.0009	0
32 Period State Probability Distribution									
$e_2 = 1$	4.30-09			7.32E-09			1.05E-12		
$e_2 = 2$	2.63E-05	0.0012		1.57E-05	0.00086		6.28E-07	1.97E-05	0
$e_2 = 3$	0.0642	0.1458	0.7888	0.0159	0.1556	0.8277	0.9955	0.0019	0.0026
Expected Welfare Outcomes									
	TS	**CS**	**PS**	**TS**	**CS**	**PS**	**TS**	**CS**	**PS**
PDV	-38.342	-	-	-38.742	-56.169	17.427	-40.702	-40.702	0
4 period	-2.051	-	-	-2.622	-2.797	0.174	-1.492	-1.492	0
32 period	-1.006	-	-	-0.959	-2.621	1.661	-1.481	-1.481	0

Figure C.2: Expected Buyer Surplus (CS) and Expected Producer Surplus as Functions of τ in a One-Shot $M = 3$ Model Where Seller Costs equal Production Costs.



C.3 Comparative Statics for $\rho = 0.02$ and $\delta = 0.058$: $M = 3$ Model Parameters with Multiple Equilibria.

We consider $\rho = 0.02$ and $\delta = 0.058$. $\rho = 0.02$ implies that production costs fall from 10 to 0.2 when know-how increases from state 1 to 2, i.e., learning-by-doing effects are extreme and sellers in states 2 and 3 have almost the same costs, but a state 2 seller is at risk of experiencing a very large cost increase (and possibly a cost disadvantage) if it does not make a sale. As δ is not too large, the social planner would prefer to maintain two low cost firms, and $D_1^{SP}(2,3) \approx 0.8$ so that both firms are very likely to be in the highest know-how state in the next period. $\rho = 0.02$ and $\delta = 0.058$ support multiple equilibria if $\tau \leq 0.202$.

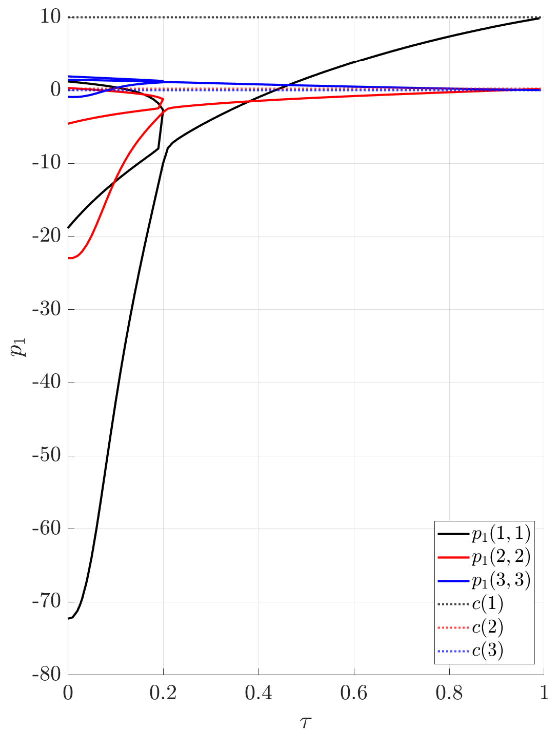
Figure C.3 shows the values of equilibrium prices in symmetric states, HHI^{32}, TS^{PDV}, and the PDVs of AB and AD incentives along τ-homotopy paths that start from the three equilibria that are identified when $\tau = 0$. One homotopy path continues across the figure, whereas the other path connects two $\tau = 0$ equilibria on a loop.

The $\tau = 0$ equilibrium that is on the path that goes across the figure has a pronounced "diagonal trench", in the language used by BDKS, as prices are very low in all symmetric states, equilibrium concentration is high and AD incentives are extremely large. Intuitively, pricing in symmetric states is so destructive of value that the leader has a very strong incentive to not allow the laggard to make sales in states where it might catch up, and the laggard has limited incentives to catch-up because fierce competition will ensure. For example, equilibrium prices in state (3,1) are 6.1 and 10.5. This equilibrium is not efficient, as buyers are unlikely to have two low cost options. On this path, TS^{PDV} does display an increasing-then-decreasing pattern but HHI^{32} has an unusual shape: there is a small initial increase, then a decrease, followed by an increase again. Dynamic incentives increase slightly as τ increases from zero, but they then decrease sharply, as the increases in τ reduces the value of the leader, whose lead is expected to last a long time.

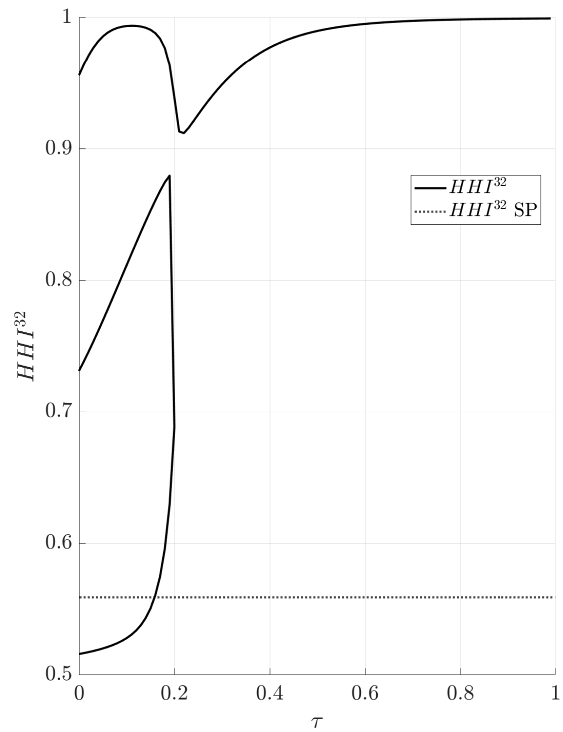
The loop connects $\tau = 0$ equilibria where pricing in symmetric states is somewhat less aggressive, e.g. prices at or above cost in state (3,3). As a result, market structure is more likely to be symmetric, efficiency is higher and concentration is somewhat lower. However, the nature of the loop makes summarizing comparative statics more complicated: for example, on one part of the loop TS^{PDV} increases as τ increases from zero, and on the other

Figure C.3: Prices in Symmetric States, Concentration, Welfare Dynamic Incentives as a Function of τ in $M = 3$ Model with $\rho = 0.02$ and $\delta = 0.058$.

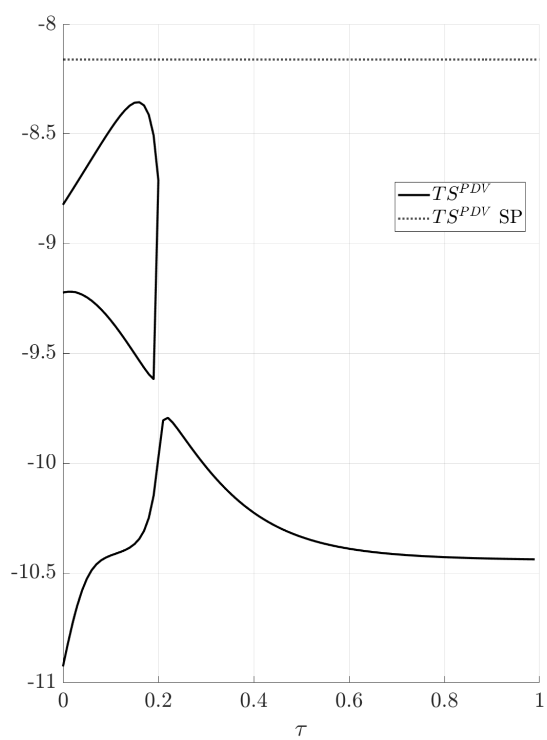
(a) Prices in Symmetric States



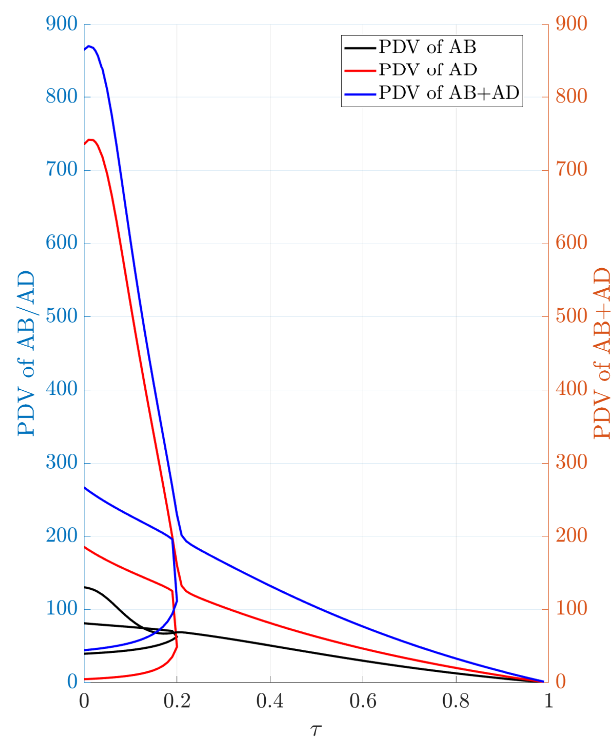
(b) HHI^{32}



(c) TS^{PDV}



(d) Dynamic Incentives



part it decreases, and then, depending on the direction of travel it then either increases or decreases. When we take the maximum TS^{PDV} across equilibria, we do find an increasing-then-decreasing pattern, but, if we move to $\delta = 0.059$, which also support multiplicity, the loop extends not quite as far to the right, and the pattern fails as, when the loop is eliminated, the maximum drops to the lower path where concentration is increasing.

For the $M = 30$ model with illustrative technology parameters ($\rho = 0.75, \delta = 0.023$), we also find three equilibria when $\tau = 0$. Two of them have diagonal trenches, and they are on a loop that does not extend past $\tau = 0.07$. Based on that example, we have been asked by discussants and seminar participants whether diagonal trench equilibria are always on paths that do not continue once τ is large enough, as this would suggest one might be able to view variation in buyer bargaining power as some type of equilibrium selection device. This $M = 3$ example provides a counter-example to this conjecture, and, while diagonal trench equilibria are often eliminated, we have identified similar counterexamples when $M = 30$ as well.

D Additional Analysis for the $M = 30$ Model

D.1 Equilibria for $\tau = 0$, $\rho = 0.75$ and $\delta = 0.023$.

Table D.1 lists strategies in a subset of states for the three equilibria that exist for the illustrative technology parameters when $\tau = 0$. All equilibria have negative prices in the initial state (1,1). The B and C equilibria are characterized by a "diagonal trench" with lower prices when firms are symmetric or almost symmetric. For example, prices in (10,10) are 2.43, whereas prices in (29,10), where costs are weakly lower, are 5.39 and 5.15. The trench in equilibrium B does not extend to the highest levels of know-how, and it has slightly lower $HHI^{1,000}$, as it is more likely that the sellers will be symmetric in the long-run. Equilibrium A, with the lowest $HHI^{1,000}$, has prices that vary less with know-how once both firms reached know-how states 3 or 4. However, the leader sets lower prices when the laggard is in know-how states 1 or 2, so that, at the start of the game, it is more likely that one of the sellers will move down its cost curve more quickly, so that HHI^{32} is higher.

D.2 Comparative Statics for $\rho = 0.75$ and $\delta = 0.023$.

The text shows how equilibrium concentration, efficiency and dynamic incentives vary with τ for our $M = 3$ model. Here we look at these relationships for $m = 15$, $M = 30$, $\rho = 0.75$ and $\delta = 0.023$, which forms our main $M = 30$ example.

We also provide some comparisons to concentration and welfare outcomes with static seller behavior (i.e., the state can change, but prices are set in a negotiation where neither buyers nor sellers account for the future). The logit demand system implies that equilibria will be unique in this case. When $\tau = 1$, sellers have no dynamic incentives in the full model, so outcomes assuming static and dynamic behavior coincide.

The solid lines in Figure D.1(a) show the homotopy paths of HHI^{32} (black) and HHI^{200} (red) in the full model. The corresponding dashed lines show expected concentration when seller behavior is static. There are three equilibria in the dynamic model when $\tau = 0$. Two equilibria are on a homotopy loop path that does not extend beyond $\tau = 0.07$.

The existence of multiplicity complicates what can be said about comparative statics: for example, while the maximum of HHI^{32} monotonically increases as τ increases from zero

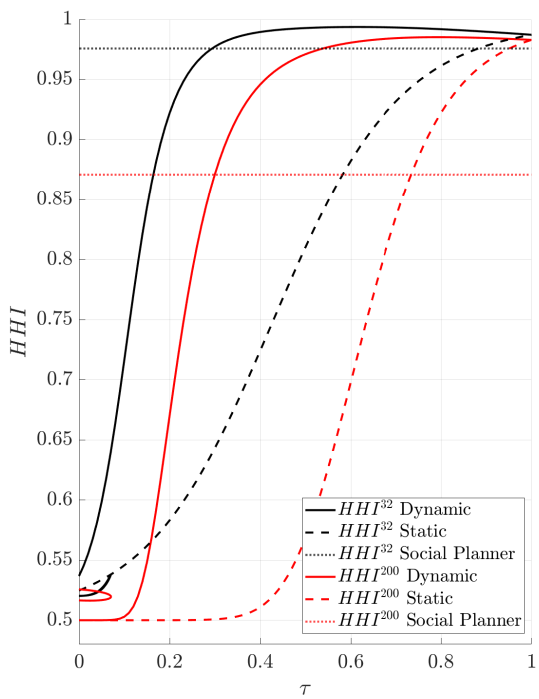
Table D.1: Equilibria for $M = 30$ Example Parameters if $\tau = 0$.

						Eqm. A		Eqm. B		Eqm. C	
						$HHI^{32} = 0.537$		$HHI^{32} = 0.520$		$HHI^{32} = 0.520$	
						$HHI^{200} = 0.500$		$HHI^{200} = 0.517$		$HHI^{200} = 0.526$	
						$HHI^{1,000} = 0.500$		$HHI^{1,000} = 0.516$		$HHI^{1,000} = 0.527$	
e_1	e_2	c_1	c_2	Δ_1	Δ_2	p_1	p_2	p_1	p_2	p_1	p_2
1	1	10.00	10.00	0.0230	0.0230	-0.54	-0.54	-1.63	-1.63	-1.61	-1.61
2	1	7.50	10.00	0.0455	0.0230	4.91	7.21	5.16	7.60	5.15	7.60
2	2	7.50	7.50	0.0455	0.0455	4.22	4.22	0.77	0.77	0.77	0.77
3	1	6.34	10.00	0.0674	0.0230	5.82	8.18	6.56	8.71	6.55	8.70
3	2	6.34	7.50	0.0674	0.0455	4.65	5.46	4.06	5.97	4.05	5.97
3	3	6.34	6.34	0.0674	0.0674	5.11	5.11	1.49	1.49	1.47	1.47
4	1	5.62	10.00	0.0889	0.0230	5.95	8.29	6.67	8.55	6.67	8.54
4	2	5.62	7.50	0.0889	0.0455	4.86	5.85	5.46	7.08	5.46	7.08
4	3	5.62	6.34	0.0889	0.0674	5.08	5.42	3.81	5.53	3.80	5.54
4	4	5.62	5.62	0.0889	0.0889	5.22	5.22	1.72	1.72	1.70	1.70
10	1	3.85	10.00	0.2076	0.0230	5.89	8.16	6.14	7.71	6.13	7.69
10	2	3.85	7.50	0.2076	0.0455	5.05	6.06	5.75	6.43	5.75	6.42
10	3	3.85	6.34	0.2076	0.0674	5.20	5.81	5.80	6.31	5.80	6.31
10	8	3.85	4.22	0.2076	0.1699	5.10	5.20	4.49	5.85	4.49	5.86
10	9	3.85	4.02	0.2076	0.1889	5.11	5.15	3.26	4.56	3.25	4.55
10	10	3.85	3.85	0.2076	0.2076	5.12	5.12	2.47	2.47	2.43	2.43
15	1	3.25	10.00	0.2946	0.0230	5.79	8.05	5.98	7.36	5.97	7.38
15	2	3.25	7.5	0.2946	0.045	5.02	5.93	5.63	6.18	5.62	6.17
15	3	3.25	6.34	0.2946	0.0674	5.22	5.74	5.67	6.02	5.67	6.01
15	10	3.25	3.85	0.2946	0.2076	5.19	5.20	5.40	5.94	5.41	5.95
15	14	3.25	3.34	0.2946	0.2780	5.23	5.21	3.46	4.44	3.43	4.44
15	15	3.25	3.25	0.2946	0.2946	5.24	5.24	3.16	3.16	3.10	3.10
16	16	3.25	3.25	0.3109	0.3109	5.28	5.28	3.24	3.24	3.18	3.18
20	20	3.25	3.25	0.3721	0.3721	5.25	5.25	3.32	3.32	3.20	3.20
22	22	3.25	3.25	0.4007	0.4007	5.25	5.25	3.44	3.44	3.26	3.26
25	25	3.25	3.25	0.4411	0.4411	5.25	5.25	3.90	3.90	3.28	3.28
27	27	3.25	3.25	0.4665	0.4665	5.25	5.25	4.62	4.62	3.34	3.34
28	28	3.25	3.25	0.4787	0.4787	5.25	5.25	4.98	4.98	3.52	3.52
29	1	3.25	10.00	0.4907	0.0230	5.79	8.05	5.63	7.62	5.57	7.46
29	2	3.25	7.50	0.4907	0.0455	5.01	5.91	5.04	5.77	5.07	5.72
29	10	3.25	3.85	0.4907	0.2076	5.23	5.17	5.35	5.17	5.39	5.15
29	15	3.25	3.25	0.4907	0.2946	5.27	5.22	5.45	5.34	5.52	5.33
29	29	3.25	3.25	0.4907	0.4907	5.25	5.25	5.22	5.22	3.98	3.98
30	1	3.25	10.00	0.5024	0.0230	5.79	8.05	5.67	7.66	5.63	7.53
30	2	3.25	7.50	0.5024	0.0455	5.01	5.91	5.10	5.84	5.12	5.81
30	10	3.25	3.85	0.5024	0.2076	5.23	5.17	5.33	5.21	5.35	5.19
30	15	3.25	3.25	0.5024	0.2946	5.27	5.22	5.42	5.35	5.45	5.33
30	29	3.25	3.25	0.5024	0.4907	5.25	5.25	5.30	5.20	4.29	4.60
30	30	3.25	3.25	0.5024	0.5024	5.25	5.25	5.27	5.27	4.77	4.77

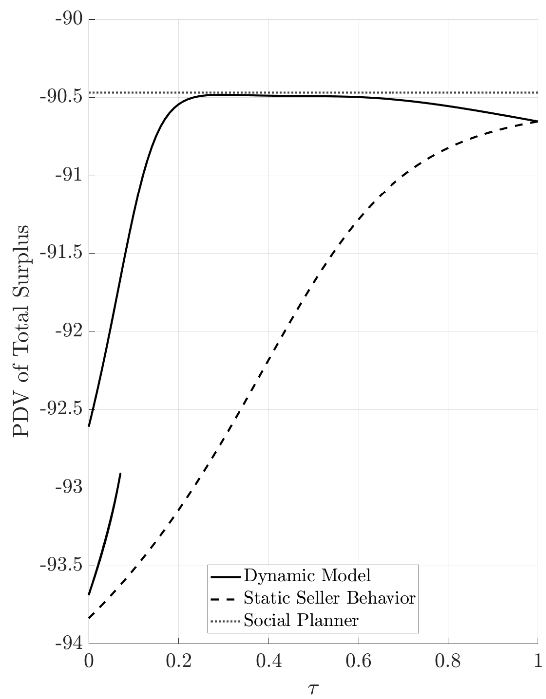
Notes: c_i, p_i, Δ_i are the marginal costs, equilibrium price and probability of forgetting for firm i. HHI^{∞} is the expected long-run value of the HHI.

Figure D.1: Equilibrium Outcomes and Welfare as Functions of τ for our $M = 30$ Model Example Parameters. Panels (a) and (b) include comparisons with expected concentration and total surplus if sellers price statically. In panel (c) only the values of dynamic incentives on the homotopy path that goes from $\tau = 0$ to $\tau = 1$ are shown.

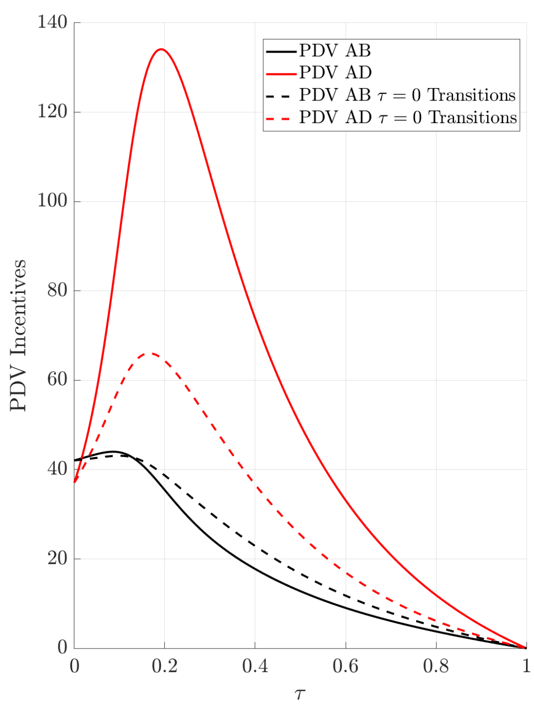
(a) HHI^{32} and HHI^{200}



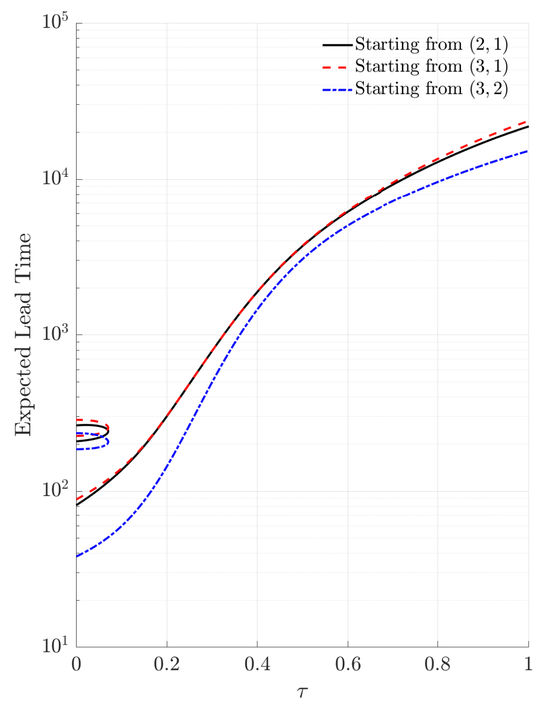
(b) TS^{PDV}



(c) PDV Dynamic Incentives



(d) Number of Periods Lead is Expected to Last



to 0.3, this is not true of the maximum of HHI^{200}, because concentration is higher for the equilibria on the loop. If we ignore multiplicity, the clear pattern is that, for low τ, increasing τ is associated with rising equilibrium market concentration, similar to our $M = 3$ example. However, for higher τ, we observe that both concentration measures slightly overshoot their $\tau = 1$ values, so that there is a slight decrease in concentration for higher levels of τ, even though the level of concentration is high. In text Figure 4(d), we would classify concentration as "almost increasing" with maximum HHI^{32} greater than 0.9 for these parameters, which is the typical pattern for ρs less than 0.85 unless δ is quite high. With static seller price-setting, concentration is close to the levels in dynamic equilibria when $\tau = 0$ and it is monotonically increasing, consistent with a larger buyer bargaining weight shrinking the margin of the leader more than the margin of the follower, and therefore increasing the probability that the leader makes the sale. There is no overshooting, consistent with overshooting reflecting dynamic incentives, which tend to increase concentration, disappearing as τ approaches 1.

Panel (b) plots TS^{PDV}. In this case, the equilibria on the loop are less efficient than the equilibria on the homotopy path across the figure, and the maximum TS^{PDV} has a monotonically increasing then monotonically decreasing pattern, with a peak that is very close to the social planner outcome, similar to our $M = 3$ example. With static seller behavior, TS^{PDV} is monotonically increasing in τ, and there is a substantial gap between the efficiency of dynamic and static equilibria for $\tau \approx 0.3$.

Panel (c) plots the PDV of sellers' AB and AD incentives, along the homotopy path that runs from $\tau = 0$ to $\tau = 1$. The solid lines show the PDVs when states are weighted by the probabilities that they are reached in equilibrium, whereas the dashed lines reflect $\tau = 0$ state weights. As in our $M = 3$ example, dynamic incentives, and particularly AD incentives, are maximized for values of τ significantly greater than zero, even though sellers' would get larger shares of surplus in any negotiation if $\tau = 0$. The comparison between the solid and dashed lines shows that the non-monotonicity in AD incentives is accentuated by how, as τ increases from zero, play shifts towards states where AD incentives are larger. As explained in the text, dynamic incentives may increase because, as τ increases, leads are expected to last longer in equilibrium. Panel (d) illustrates that the number of periods that leads are expected to last in selected low know-how states increase quickly with τ, even though the differences in costs between adjacent know-how levels are not so large with $\rho = 0.75$.

D.3 Comparative Statics for $\rho = 0.95$ and $\delta = 0.03$.

Our comparative static and application discussions focus on parameters where LBD effects are fairly strong. However, for completeness, it is interesting to understand what are the effects of the bargaining weight parameter when LBD is insignificant. In this subsection, we consider $\rho = 0.95$ and $\delta = 0.03$. These parameters also provide examples of multiple equilibria existing for some $\tau > 0$, even though the equilibrium when $\tau = 0$ is unique.

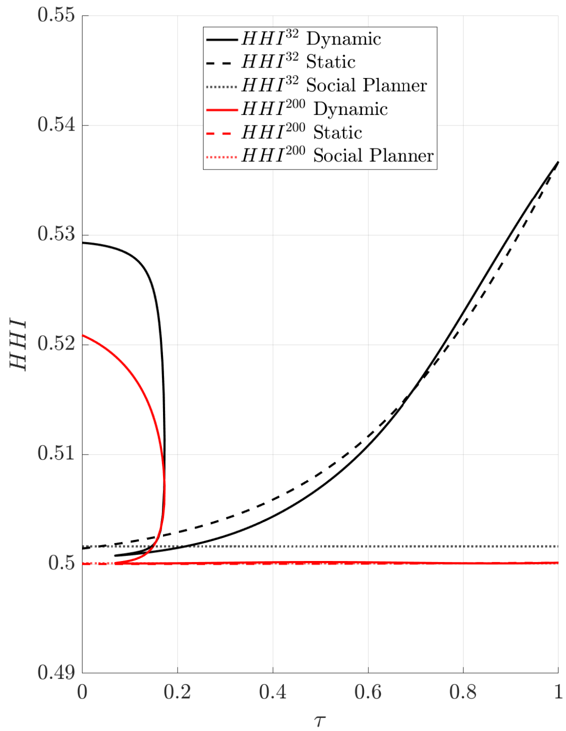
Figure D.2 replicates Appendix Figure D.1 for these new parameters. There is only one homotopy path, but there are multiple equilibria because the path bends back on itself, so that there are three equilibria for a narrow range of $0.07 \leq \tau \leq 0.175$. The unique equilibrium with $\tau = 0$ has concentration a little above the socially optimal level, and the level that would be generated by static seller pricing. However, once the path has dropped down, concentration under the dynamic and static equilibria almost coincide for all τ.

The remaining figures show what happens to TS^{PDV}, the PDV of AB and AD incentives, and the expected number of periods that a lead is expected to last in selected low know-how states. TS^{PDV} shows an increasing-then-decreasing pattern, as in our other examples, but the general pattern is that dynamic incentives decline with τ, and, in the region where we observe concentration declining, lead lengths also decline in τ. This suggests that some of the results we highlight in the text may depend on assuming that LBD effects are relatively large. We can further investigate this point by re-examining some of our applications assuming these alternative technology parameters.

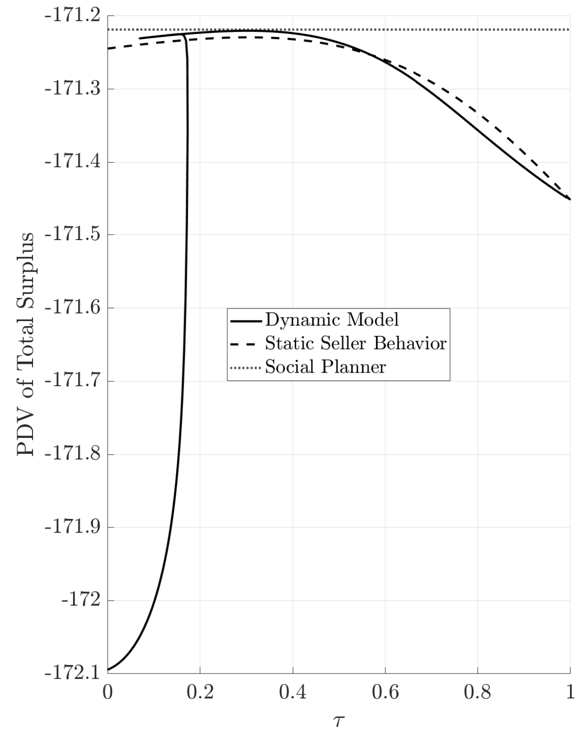
Figure D.3(a) shows the optimal subsidies that would implement the social planner solution as a function of τ. When $\tau = 0$, the subsidies are much smaller in scale than for the illustrative $M = 30$ technology parameters. This is, of course, consistent with no subsidy equilibrium concentration being closer to the socially optimal level. Even though equilibrium concentration is too high when $\tau = 0$, laggards are taxed when they make a sale. This reflects how, given socially optimal sales probabilities in other states, the laggard would be too likely to make a sale without a tax. Panel (b) shows that the $\tau = 0$ subsidy scheme lowers welfare for sure if $\tau \geq 0.175$. For $0.08 \leq \tau \leq 0.175$ the scheme may increase or decrease welfare depending on which no-subsidy equilibrium is played. Therefore, the conclusion that

Figure D.2: Concentration, Welfare, Dynamic Incentives and Lead Length as a Function of τ for $M = 30$, $\rho = 0.95$ and $\delta = 0.03$.

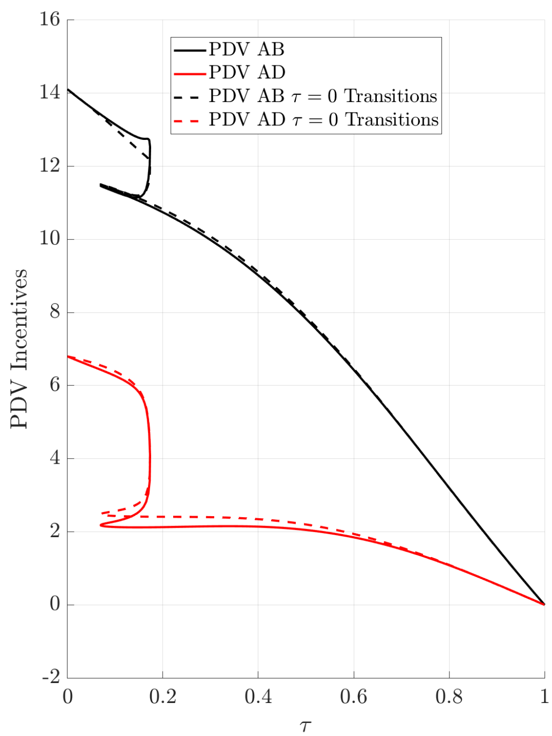
(a) HHI^{32} and HHI^{200}



(b) TS^{PDV}



(c) PDV Dynamic Incentives



(d) Number of Periods Lead is Expected to Last

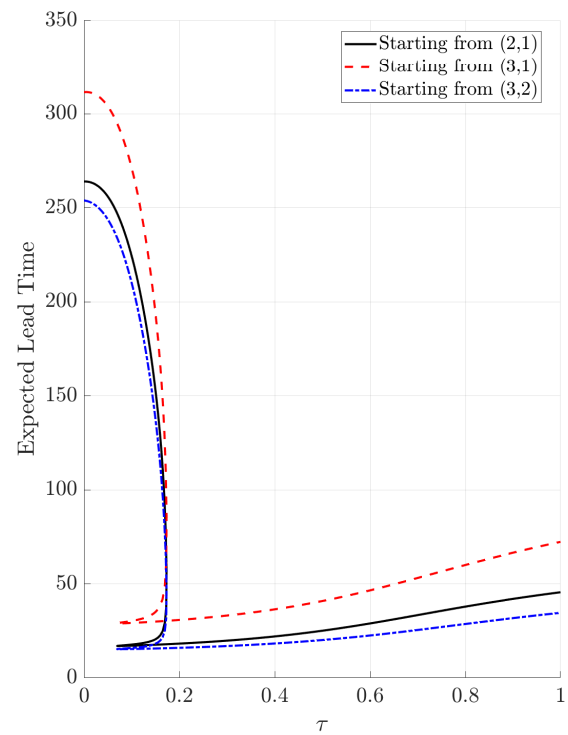
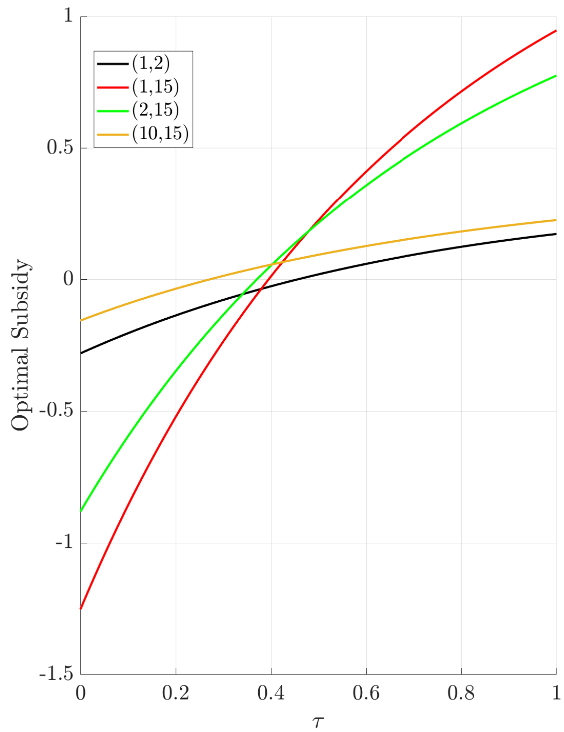
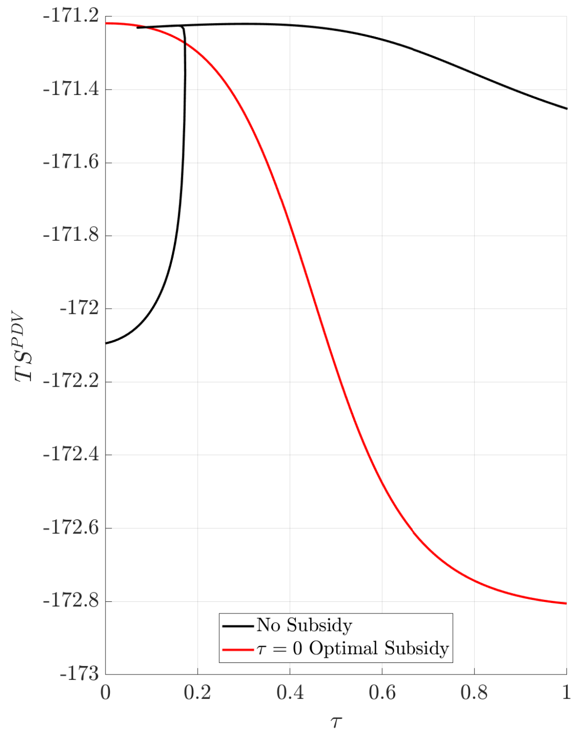


Figure D.3: Subsidies and Policies to Promote Competition as a Function of τ for $M = 30$, $\rho = 0.95$ and $\delta = 0.03$. Subsidies are given to the laggard when it makes a sale. This analysis assumes that policies are introduced at the start of the industry's life.

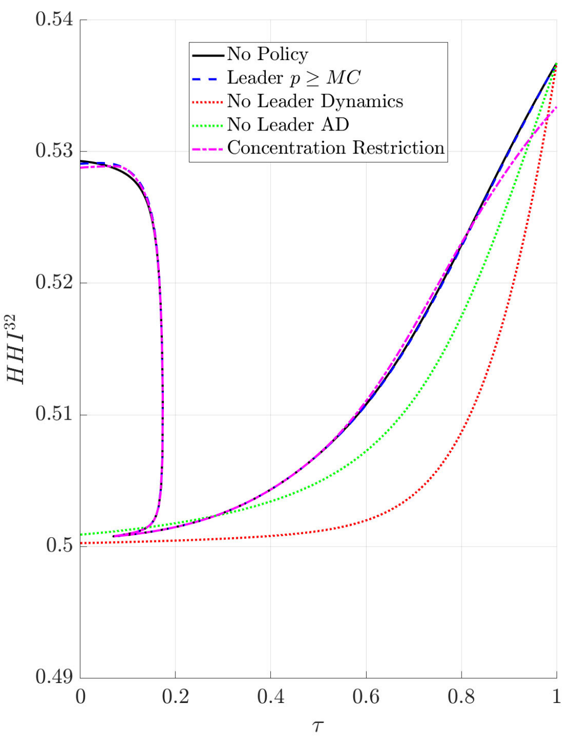
(a) Optimal Subsidies in Selected States



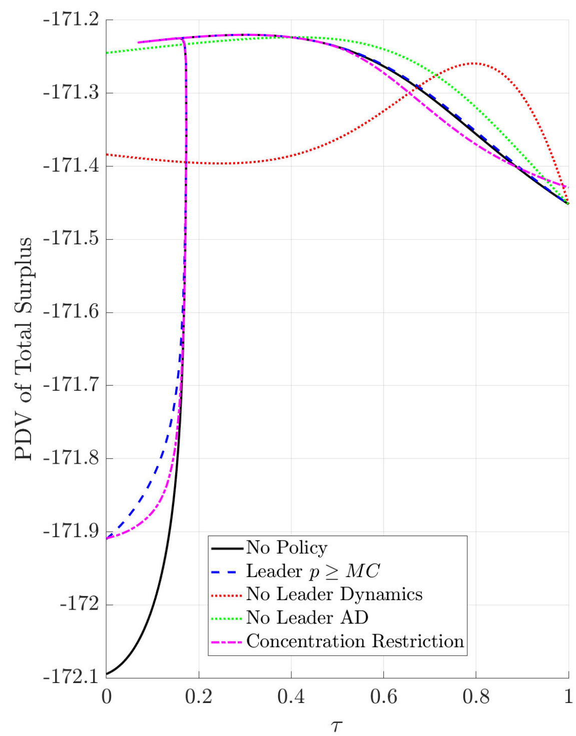
(b) Welfare and $\tau = 0$ Optimal Subsidies



(c) Policies Promoting Competition: HHI^{32}



(d) Policies Promoting Competition: TS^{PDV}

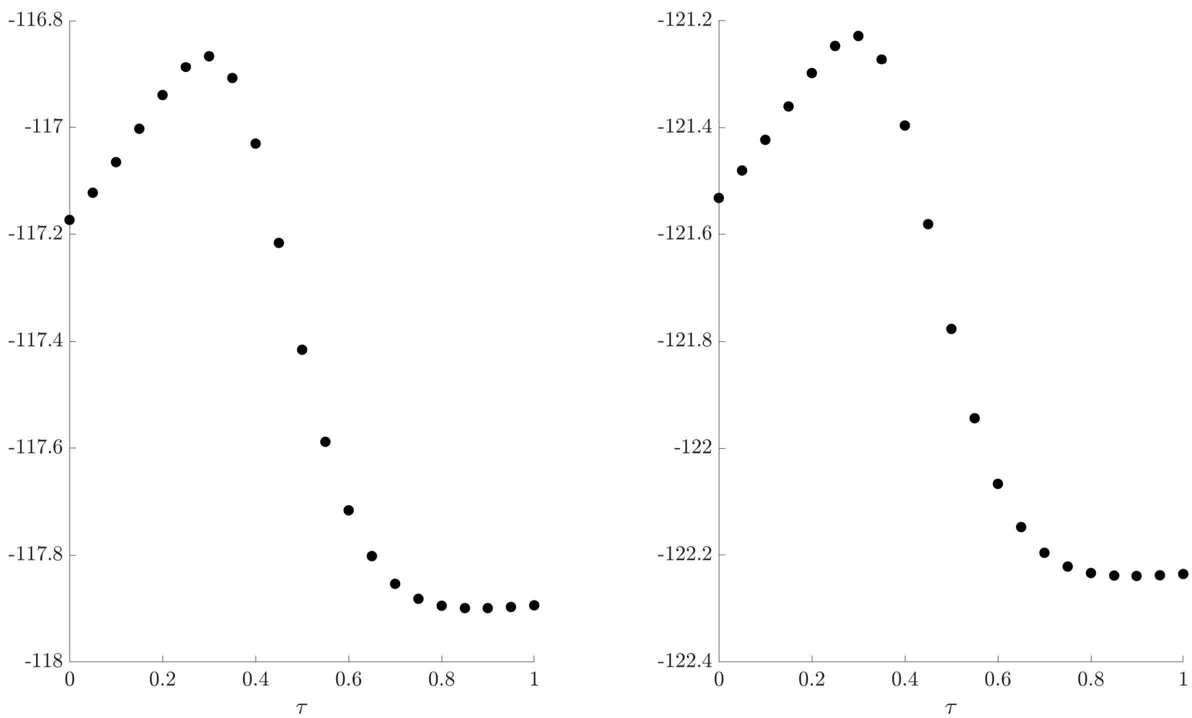


subsidies that would maximize welfare when $\tau = 0$ can lower welfare for values of τ that are greater than zero, but small, remains.

Panels (c) and (d) show the effects of our stylized policies to promote competition, assuming that they are introduced at the start of the industry. As equilibrium concentration is very low until τ approaches 1, the concentration restriction has almost no effect on welfare or concentration until τ is large. The incentive policies increase welfare when $\tau \approx 0$, and they also increase welfare for $\tau > 0.7$.

Figure D.4: Welfare as a Function of τ for Two Technologies where the Relationship is Not “Monotonically Increasing-then-Monotonically Decreasing”. The dots indicate the level of TS^{PDV} at the 21 values of τ used to identify the existence of multiple equilibria. There are small increases in TS^{PDV} for large τ.

(a) $\rho = 0.83$, $\delta = 0.01$ (b) $\rho = 0.84$, $\delta = 0.05$



D.4 TS^{PDV} Comparative Statics for $\rho \approx 0.83$.

Text Figure 4 shows that TS^{PDV} has a monotonically increasing-then-monotonically decreasing pattern for almost all technology parameters which support a unique equilibrium for all τ. However, there is a small range of technologies where δ is small and $\rho \approx 0.825$ where this pattern is classified as failing. Figure D.4 shows what the relationship actually is for two examples of these parameters.

In both cases, we observe an increasing-then-decreasing pattern up to $\tau = 0.85$. After this point, TS^{PDV} increases by very small amounts as τ increases further. As most of our focus is on changes in outcomes for lower values of τ, these examples, while illustrating that our comparative statics are not completely general, do not affect our summary that an increasing-then-decreasing pattern is standard.

E Supporting Materials for Extensions and Applications.

E.1 Introducing an Outside Good and Varying σ.

We follow Besanko, Doraszelski, Kryukov, and Satterthwaite (2010) (BDKS) in assuming that there is no outside good and that σ, which controls the degree of product differentiation, equals 1. We briefly examine here whether our conclusions about the relationships between the bargaining weight τ and equilibrium outcomes would change. We use our $M = 3$ and $M = 30$ example parameters.

E.1.1 Outside Good.

When the buyer is able to choose not to purchase, sellers face additional competition. One intuition would be that this will weaken dynamic incentives, as it may reduce a firm's payoffs when it has a large lead. On the other hand, it might strengthen these incentives if its primary effect is to make it more difficult for a laggard to catch-up.

We introduce an outside good by assuming that, in every period, buyers have a third option, with indirect utility $v - p_0 + \varepsilon_0$, where p_0 is an exogenous parameter that we can use to control the attractiveness of the outside good. Besanko, Doraszelski, and Kryukov (2014), who consider a model where only a single seller may be active, allow an outside good with a baseline $p_0 = 10$.

Figure E.1(a) shows the equilibrium values of HHI^{32} for $\tau = 0$, 0.2 and 0.5 as a function of p_0. Figure E.1(c) and (e) show the PDV of AB and AD incentives. The BDKS model can be viewed as the limiting case where $p_0 \to \infty$ (i.e., we extend the right-hand edge of the figure further to the right). The effects of the outside good are small unless $p_0 < 7$, in which case dynamic incentives increase and concentration (reflecting the dominance of one seller over its rival) increases slightly. However, for $p_0 \approx 10$, all measures are very similar to those when an outside good is assumed not to exist, and introducing an outside good has little effect on the comparisons between different τs.

Panels (a), (c) and (e) of Figure E.2 shows a similar set of analyses for our $M = 30$ example. The patterns are similar, although the levels of concentration and dynamic incentives

Figure E.1: HHI^{32} and the PDV of Dynamic Incentives as a Function of the Exogenous Price of an Outside Good (panels a, c, and e) and Product Differentiation (b, d and f) for Different τs in our $M = 3$ Example.

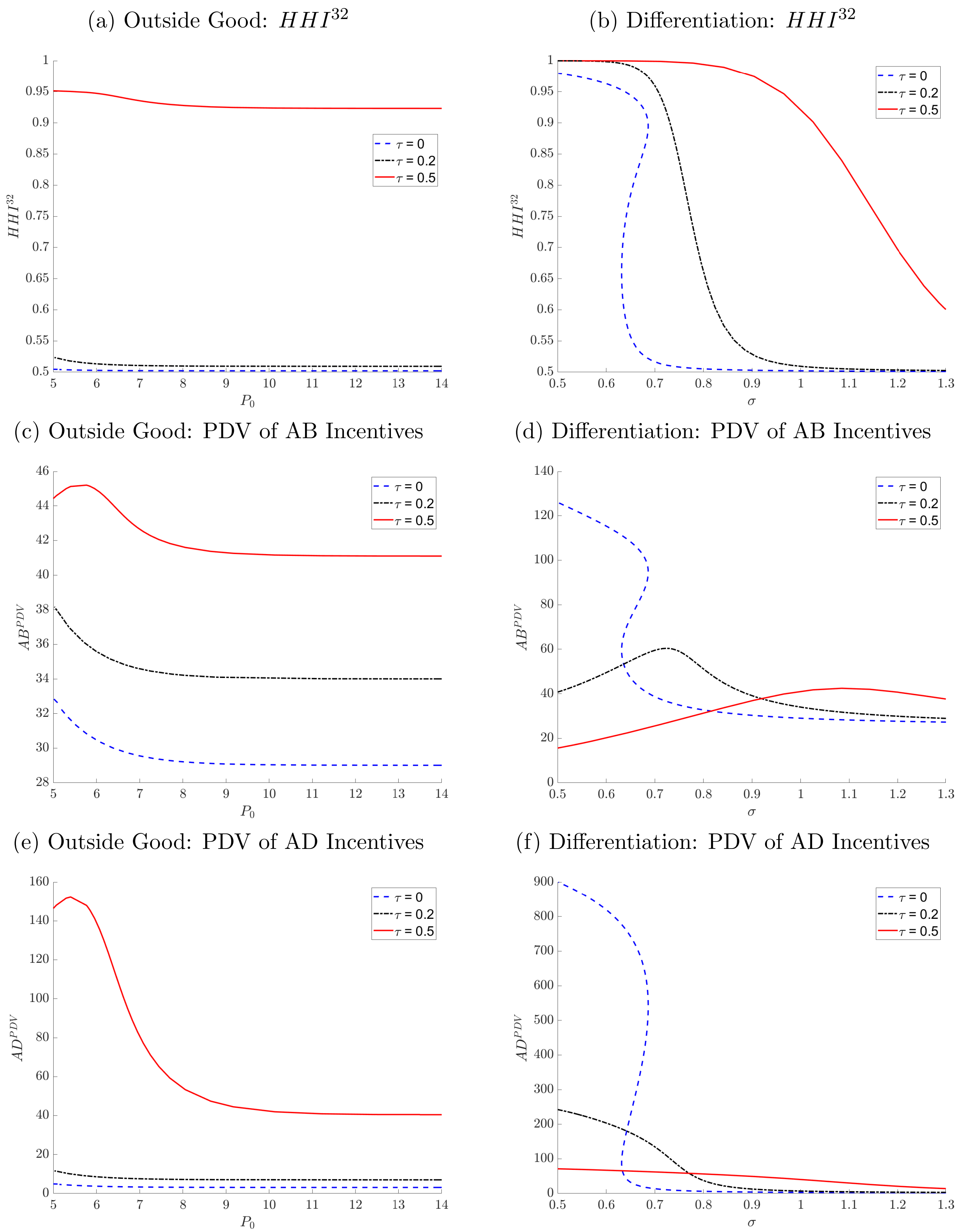
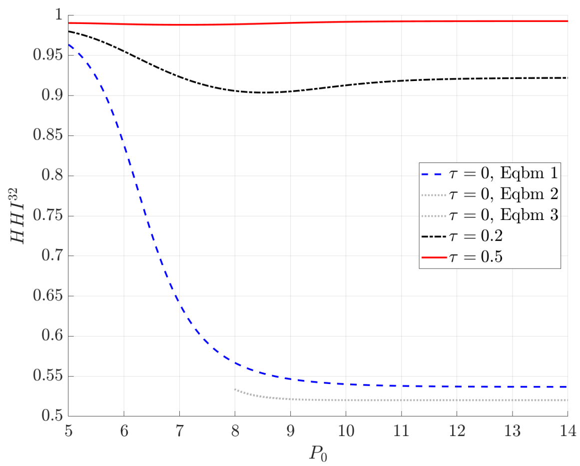
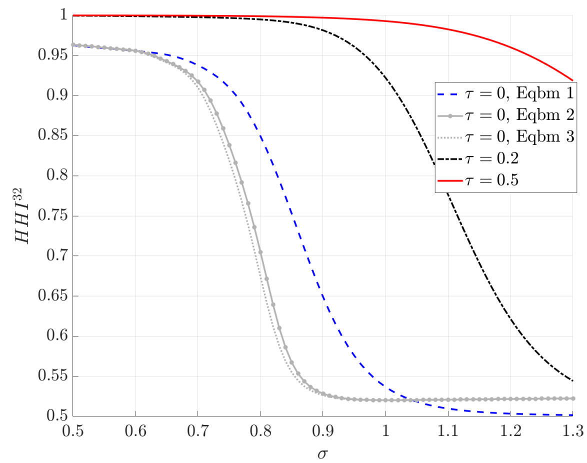


Figure E.2: HHI^{32} and the PDV of Dynamic Incentives as a Function of the Exogenous Price of an Outside Good (panels a, c, and e) and Product Differentiation (b, d and f) for Different τs in our $M = 30$ Example.

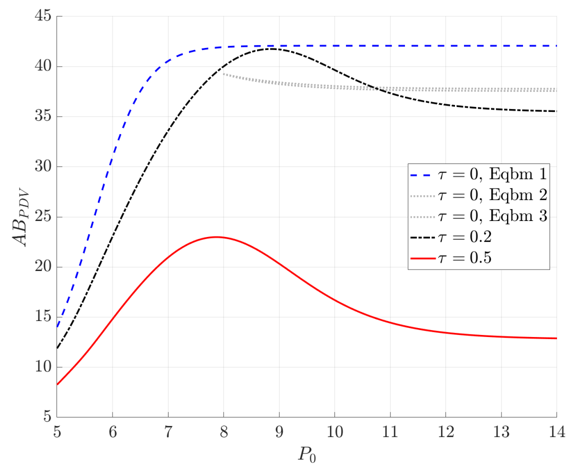
(a) Outside Good: HHI^{32}



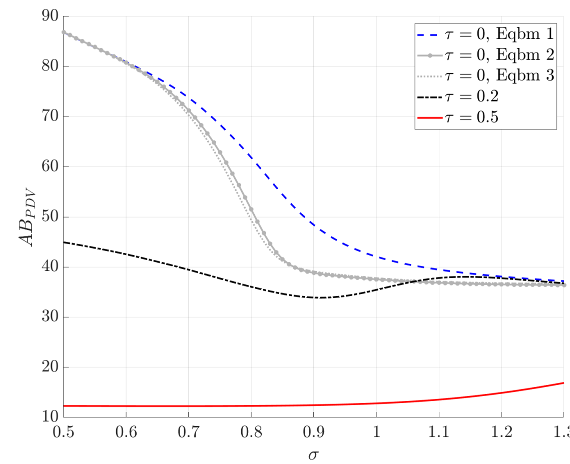
(b) Differentiation: HHI^{32}



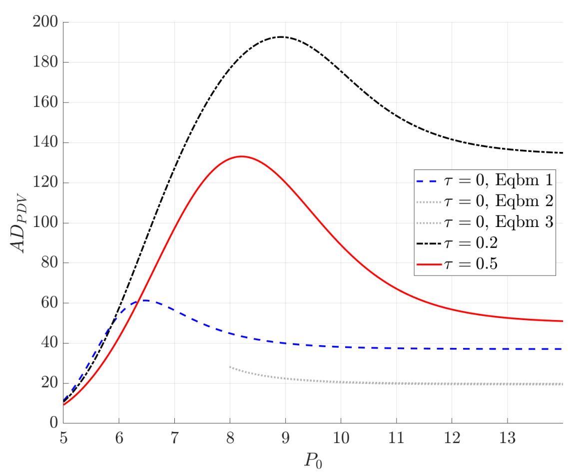
(c) Outside Good: PDV of AB Incentives



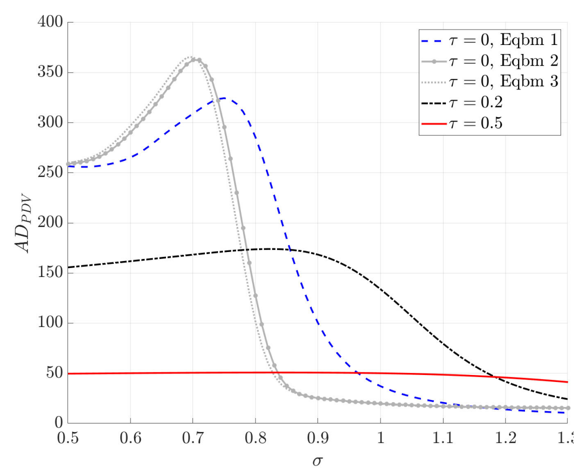
(d) Differentiation: PDV of AB Incentives



(e) Outside Good: PDV of AD Incentives



(f) Differentiation: PDV of AD Incentives



change by larger amounts as the outside good becomes more competitive in the model with the larger state space, and the order of AD incentives with respect to τ changes when the outside good is really quite competitive even when seller costs are low.

E.1.2 Changing Product Differentiation.

As σ increases, it becomes more likely that, in any period, the buyer will have a strong preference for one of the sellers, so that seller competition is softened. This will tend to make it more likely that purchases will be evenly split across sellers, although the expectation of future sales and softened competition could increase a firm's incentive to lower its own costs.

Figures E.1 panels (b), (d) and (f) show the same statistics as σ is varied for the $M = 3$ model, assuming that there is no outside good. Increasing σ from 1 lowers concentration, but does not dramatically change how giving buyers bargaining power affects outcomes. On the other hand, reducing σ to 0.8, or lower, causes equilibrium concentration to increase sharply and can introduce multiple equilibria when $\tau = 0$, illustrated by the $\tau = 0$ path bending back on itself. Higher concentration is associated with leads lasting longer when $\tau = 0$, which tends to lead to dynamic incentives monotonically declining in τ.

For the $M = 30$ example, Figure E.2, the changes are qualitatively similar, although dropping σ from 1 to 0.8, would mean that AD incentives would be larger when $\tau = 0$ than $\tau = 0.25$ or $\tau = 0.5$.

E.2 Forward-Looking Buyers.

In this section we explain how the math of the model changes when we allow for symmetric repeat buyers, following Sweeting, Jia, Hui, and Yao (2022) (SJHY), before showing the results for the $M = 30$ model.

Formulation. We model repeat buyers by assuming that any buyer expects a fraction b^p of future buyer surplus. One way to think about this is that there are a symmetric pool of potential buyers and that each period nature chooses, with replacement, one of these potential buyers to be that period's buyer. The baseline model corresponds to $b^p = 0$ (e.g., an infinite number of potential buyers), and below we will consider $b^p = 0.5$ (two potential buyers) and $b^p = 0.2$ (five potential buyers).

In the formulation where we define the equilibrium in terms of prices and values, this introduces an additional set of values, V^B, which equals the value of a representative potential buyer before nature has drawn the actual buyer. The probability that the chosen buyer purchases from seller k, $D_k(\mathbf{e})$, given negotiated prices, is

$$D_k(\mathbf{e}) = \frac{\exp\left(\frac{v-p_k(\mathbf{e})+\mu_k^B(\mathbf{e})}{\sigma}\right)}{\exp\left(\frac{v-p_1(\mathbf{e})+\mu_1^B(\mathbf{e})}{\sigma}\right)+\exp\left(\frac{v-p_2(\mathbf{e})+\mu_2^B(\mathbf{e})}{\sigma}\right)},$$

where $\mu_k^B(\mathbf{e})$, the buyer continuation value when it purchases from k, is defined below, and

$$V^{B*}(\mathbf{e}) - b^p\sigma\log\left(\sum_{k=1,2}\exp\left(\frac{v-p_k^*(\mathbf{e})+\mu_k^B(\mathbf{e})}{\sigma}\right)\right) - (1-b^p)\sum_{k=1,2}D_k^*(\mathbf{e})\mu_k^B(\mathbf{e}) = 0 \quad (38)$$

where

$$\mu_k^B(\mathbf{e}) = \beta\sum_{\forall e'_{1,t+1}|e_{1,t}}\sum_{\forall e'_{2,t+1}|e_{2,t}} V^{B*}(e'_{1,t+1},e'_{2,t+1})\Pr(e'_{1,t+1}|e_{1,t},k)\Pr(e'_{2,t+1}|e_{2,t},k).$$

The formulae for pricing first-order conditions are the same as before except that $CS(p_1,p_2,\mathbf{e}) = \sigma\log\left(\sum_{k=1,2}\exp\left(\frac{v-p_k(\mathbf{e})+\mu_k^B(\mathbf{e})}{\sigma}\right)\right)$ and $CS(p_2,\mathbf{e}) = v - p_2(\mathbf{e}) + \mu_2^B(\mathbf{e})$.

We can also easily incorporate repeat buyers into our alternative formulation.

$$\sigma\log\left(\frac{1}{D_1(\mathbf{e})}-1\right) - p_1(\mathbf{e}) + p_2(\mathbf{e}) + \mu_1^B(\mathbf{e}) - \mu_2^B(\mathbf{e}) = 0. \quad (39)$$

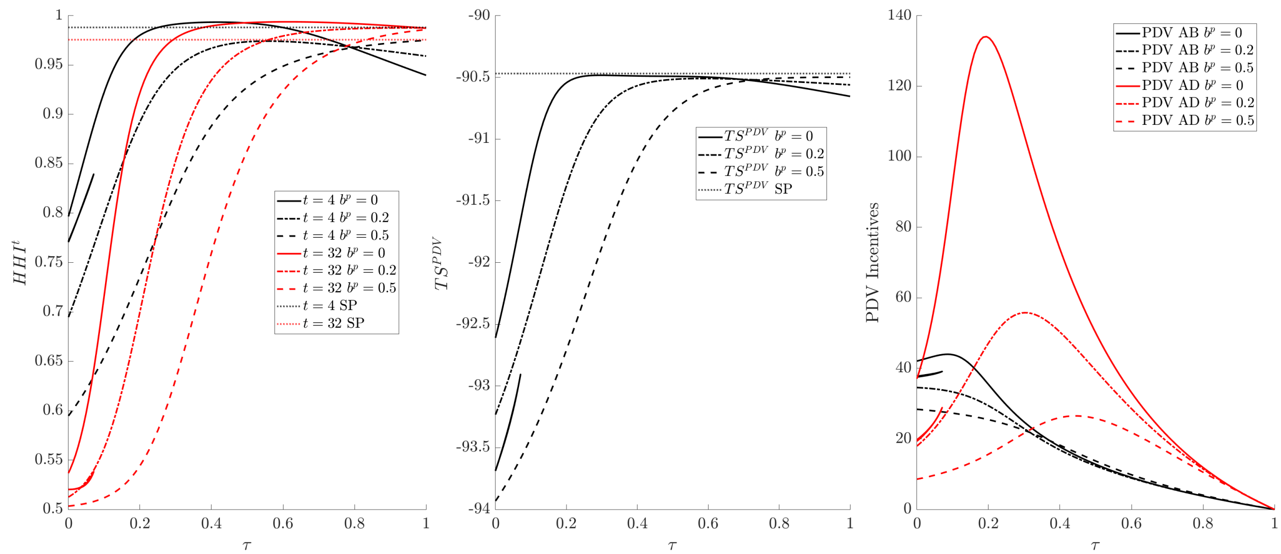
If $b^p > 0$, the differences in buyer continuation values are $\boldsymbol{\mu_1^B} - \boldsymbol{\mu_2^B} = \beta(\mathbf{Q_1}-\mathbf{Q_2})\mathbf{V^B}$, and

$$\mathbf{V^B} = b^p\left(\mathbf{I} - \beta\sum_{k=1,2}\mathbf{D_k}\circ\mathbf{Q_k}\right)^{-1}\sum_{k=1,2}\left[\mathbf{D_k}\circ\left(\sigma\log\frac{1}{\mathbf{D_k}} + v - \mathbf{p_k}\right)\right]. \quad (40)$$

so we can also substitute for the continuation buyer values with choice probabilities. Therefore adding repeat buyers does not increase the number of equations in the alternative

Figure E.3: Concentration, Welfare and Dynamic Incentives as a Function of τ for $M = 30$ Model with $\rho = 0.75$, $\delta = 0.023$.

(a) HHI^t (b) TS^{PDV} (c) PDV Dynamic Incentives



formulation.

Effects of Forward-Looking Buyers in the $M = 30$ Example. Text Figure 5 shows how the relationships between concentration, efficiency and dynamic incentives and τ in our $M = 3$ example change when we assume $b^p = 0.2$, equivalent to five symmetric repeat and forward-looking buyers, and $b^p = 0.5$, equal to two symmetric such buyers. Figure E.3 shows the same relationships for our $M = 30$ example.

Consistent with SJHY's findings, concentration and sellers' dynamic incentives (for $\tau < 1$) decrease, and multiple equilibria are eliminated, as b^p is increased. However, even though $b^p = 0.2$ and $b^p = 0.5$ imply significant concentration on the buyer side of the market, the qualitative relationships between τ and the three outcome measures do not change much, except that, with $b^p = 0.5$, TS^{PDV} is maximized, at close to the socially optimal level, at $\tau = 1$.

E.3 Additional, and Multiproduct, Sellers: General Formulation of the Model.

Our baseline model assumes single product duopolists. However, our extensions allow for more sellers and for a merger creating a multiproduct seller.

Consider an industry where K products are sold by N firms. The products are indexed by the set $\mathcal{K} = \{1, 2, \dots, K\}$, and the firms are indexed by the set $\mathcal{N} = \{1, 2, \dots, N\}$. Selling firm $n \in \mathcal{N}$ owns a set A_n of products, and $\bigcup_{n\in\mathcal{N}} A_n = \mathcal{K}$.

Bargaining First-Order Condition with Multiproduct Firms. The buyer bargains over a set of prices, $\mathbf{p}_{A_n} = \{p_k : k \in A_n\}$, with seller $n \in \mathcal{N}$. Negotiated prices will be the solutions to the following problem

$$\begin{aligned}\max_{\mathbf{p}_{A_n}} & \left[\sigma \log \sum_k \exp\left(\frac{v - \alpha p_k}{\sigma}\right) - \sigma \log \sum_{k\notin A_n} \exp\left(\frac{v - \alpha p_k}{\sigma}\right)\right]^{\tau} \\ & \times \left[\sum_{k\in A_n} D_k(\mathbf{p})(p_k - c_k + \mu_k^n) + \sum_{k\notin A_n} D_k(\mathbf{p})\mu_k^n - \sum_{k\notin A_n} D_k^{-n}(\mathbf{p}_{-n})\mu_k^n\right]^{1-\tau},\end{aligned} \tag{41}$$

where μ_k^n is the continuation value of seller n conditional on the buyer purchases product k, μ_k^B is the continuation value of the buyer conditional on the buyer purchases product k, and

$$D_k(\mathbf{p}) = \frac{\exp\left(\frac{v-\alpha p_k}{\sigma}\right)}{\sum_j \exp\left(\frac{v-\alpha p_j}{\sigma}\right)}, D_k^{-n}(\mathbf{p}_{-n}) = \frac{\exp\left(\frac{v-\alpha p_k}{\sigma}\right)}{\sum_{j\notin A_n} \exp\left(\frac{v-\alpha p_j}{\sigma}\right)}.$$

Note that for $k \notin A_n$, $D_k(\mathbf{p}) = (1 - D_{A_n}(\mathbf{p}))D_k^{-n}(\mathbf{p}_{-n})$, where $D_{A_n}(\mathbf{p})$ denotes the total market share of firm n:

$$D_{A_n}(\mathbf{p}) = \sum_{k\in A_n} D_k(\mathbf{p}).$$

The bargaining problem can be rewritten as

$$\max_{\mathbf{p}_{A_n}} \left[-\sigma \log\left(1 - D_{A_n}(\mathbf{p})\right)\right]^{\tau} \left[\sum_{k\in A_n} D_k(\mathbf{p})(p_k - \widehat{c_k})\right]^{1-\tau}, \tag{42}$$

where $\widehat{c_k}$ denotes the "effective" or continuation-adjusted marginal cost. That is,

$$\widehat{c_k} = c_k - \mu_k^n + \sum_{j \notin A_n} D_j^{-n}(\mathbf{p}_{-n})\mu_j^n. \tag{43}$$

The FOC for p_j, with $j \in A_n$, is

$$-\tau \sum_{k \in A_n} D_k(\mathbf{p})(p_k - \widehat{c_k}) + (1-\tau)\left[\frac{\sigma}{\alpha} - (p_j - \widehat{c_j}) + \sum_{k \in A_n} D_k(\mathbf{p})(p_k - \widehat{c_k})\right] \log \frac{1}{1 - D_{A_n}} = 0. \tag{44}$$

Corollary E.1 *The markup over effective marginal cost $p_k - \widehat{c_k}$ is the same across products within one firm. Specifically, for all $k \in A_n$,*

$$p_k - \hat{c}_k = \Phi(D_{A_n}) := \frac{\frac{\sigma}{\alpha}(1-\tau) \log \frac{1}{1 - D_{A_n}}}{\tau D_{A_n} + (1-\tau)(1 - D_{A_n}) \log \frac{1}{1 - D_{A_n}}}. \tag{45}$$

This corollary extends the well-known property in static models with multinomial logit demand that a multiproduct firm will set its price so that there is an identical markup on all of its products.

The equilibrium prices p_k, $k \in \mathcal{K}$, and firm values VS_n, $n \in \mathcal{N}$, can be solved from (9) together with

$$V_n^S = \sum_{k \in A_n} D_k(\mathbf{p})(p_k - c_k) + \sum_{k \in \mathcal{K}} D_k(\mathbf{p})\mu_k^n,$$

or equivalently,

$$V_n^S = \sum_{k \in A_n} D_k(\mathbf{p})(p_k - \hat{c}_k) + \sum_{k \notin A_n} \mu_k^n. \tag{46}$$

There are $(K + N) \times M^K$ equations in total (K prices and N firm values for each one of the M^K states).

Choice Probability Formulation of Equilibrium Equations. It is also possible to formulate the equilibrium in terms of buyer choice probabilities. Specifically, from (46) we have that

$$\mathbf{V}_n^S = \left(\mathbf{I} - \beta \sum_{k \notin A_n} \mathbf{Q}_k\right)^{-1} \left[\mathbf{D}_{A_n} \circ \mathbf{\Phi}(\mathbf{D}_{A_n})\right].$$

Plugging this into (9) yields that

$$\mathbf{p}_k = \mathbf{\Phi}(\mathbf{D}_{A_n}) + \mathbf{c}_k - \beta \left(\mathbf{Q}_k - \sum_{j \notin A_n} \mathbf{D}_j^{-n} \circ \mathbf{Q}_j\right) \left(\mathbf{I} - \beta \sum_{k \notin A_n} \mathbf{Q}_k\right)^{-1} \left[\mathbf{D}_{A_n} \circ \mathbf{\Phi}(\mathbf{D}_{A_n})\right].$$

Consequently, prices and firm values can be pinned down from the *aggregate* market shares at firm level, D_{A_n}. So to characterize the equilibrium, it suffices to solve for D_{A_n} from the following equation:

$$D_{A_n}(\mathbf{p}) = \sum_{k \in A_n} \frac{\exp\left(\frac{v - \alpha p_k}{\sigma}\right)}{\sum_j \exp\left(\frac{v - \alpha p_j}{\sigma}\right)}.$$

There are $N \times M^K$ equations in total (N firm shares for each one of the M^K states).

Optimal Subsidies. Given the social planner's market shares D_k^{SP}, the optimal subsidies s_k and corresponding prices p_k can be solved using the following two equations.

$$\begin{aligned} D_k^{SP} &= \frac{\exp\left(\frac{v - \alpha p_k}{\sigma}\right)}{\sum_j \exp\left(\frac{v - \alpha p_j}{\sigma}\right)}, \\ p_k - \tilde{c}_k &= \frac{\frac{\sigma}{\alpha}(1 - \tau) \log \frac{1}{1 - D_{A_n}}}{\tau D_{A_n} + (1 - \tau)(1 - D_{A_n}) \log \frac{1}{1 - D_{A_n}}}, \end{aligned}$$

where

$$\begin{aligned} \tilde{c}_k &= c_k - s_k - \mu_k^n + \sum_{k \notin A_n} D_k^{-n}(\mathbf{p}_{-n}) \mu_k^n, \\ \mu^n &= \sum_{k \in A_n} D_k^{SP}(p_k - c_k + s_k) + \sum_{k \in \mathcal{K}} D_k^{SP} \mu_k^n, \\ \mu_k^n &= \beta \sum_{\mathbf{e}'} \Pr(\mathbf{e}' | \mathbf{e}, k) \mu^n(\mathbf{e}'). \end{aligned}$$

In matrix form,

$$\mu^n = (\mathbf{I} - \beta \mathbf{Q})^{-1} \left[\sum_{k \in A_n} \mathbf{D}_k^{SP} \circ (\mathbf{p}_k - \mathbf{c}_k + \mathbf{s}_k) \right].$$

E.3.1 Effects of a Horizontal Merger with Static Seller Behavior.

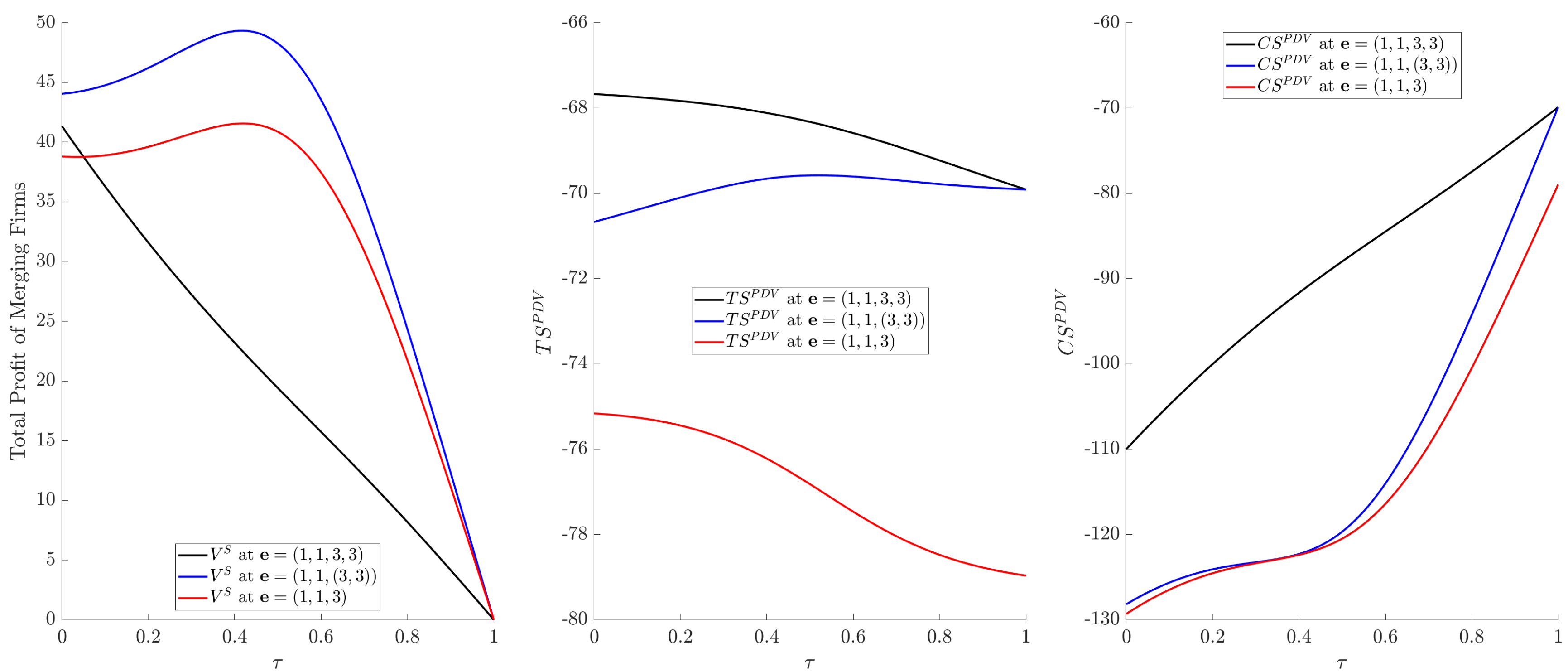
Text Section 5.4 considers the profitability and welfare effects of a horizontal merger between the leading firms in state (1,1,3,3) in an $M = 7$ and $m = 5$ version of the model with up to four firms.

It is also possible to calculate the effects when we assume static seller behavior, but the state is allowed to change. Figure E.4 shows a similar set of diagrams to text Figure 8, but now assuming static seller behavior.

Both the results when $\tau = 0$, and the relationships of outcomes to τ, are different when dynamic behavior is not allowed. For example, a merger without product elimination is profitable for all τ, but it also lowers total surplus and consumer surplus.

Figure E.4: Effects of a Horizontal Merger Between Firms with $e_3 = 3$ and $e_4 = 3$ assuming Static Seller Behavior as a Function of τ for $M = 7$, $m = 5$ for $\rho = 0.65$ and $\delta = 0.03$. Rivals' states when the merger is proposed as $e_1 = 1$ and $e_2 = 1$.

(a) Merging Firm Values (b) TS^{PDV} (c) CS^{PDV}



E.4 Subsidies.

E.4.1 Subsidies with Static Seller Behavior

The text shows the structure of optimal subsidies, and the welfare effects of imposing $\tau = 0$ optimal subsidies when buyers have some degree of bargaining power, in the dynamic model. We find that, when τ is above some low threshold, $\tau = 0$ subsidies reduce welfare relative to having no policies. As a comparison, and to understand the role of dynamics, we show here what would happen if we assume static seller behavior.

The change in the structure of optimal subsidies is greatest for $M = 30$ and small τ, as in some states, the large positive subsidies in the dynamic model are replaced by large taxes. However, the pattern that some transfers change rapidly as τ is increased from zero is maintained. With dynamic seller behavior we found that $\tau = 0$ optimal subsidies lowered efficiency for τs above some quite low thresholds. With static behavior, the pattern is different. For $M = 3$, $\tau = 0$ subsidies increase welfare until $\tau > 0.38$, and they always increase welfare in the $M = 30$ model.

E.4.2 Welfare Effects of Subsidies for Alternative Technologies

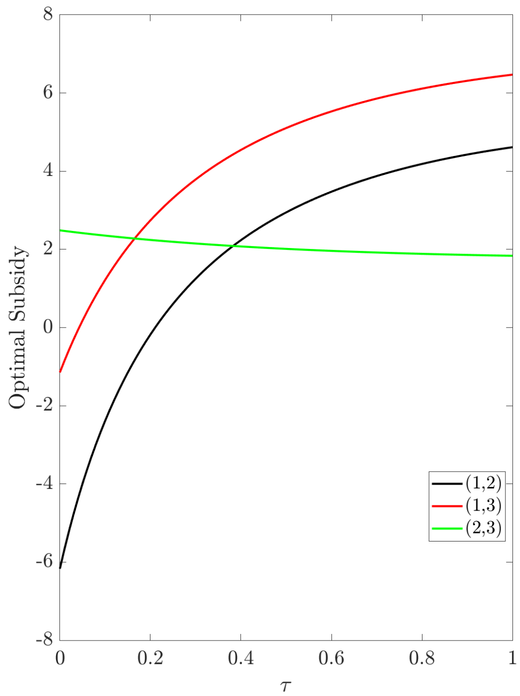
The text shows that, in the $M = 3$ example, the subsidies that would be optimal if $\tau = 0$ lower TS^{PDV}, relative to the no subsidy equilibrium, if $\tau \geq 0.06$. We try to investigate whether this result generalizes by testing whether optimal $\tau = 0$ subsidies lower TS^{PDV} for other (ρ, δ) combinations when $\tau = 0.2$.

Figure E.6(a) provides the results. The gray area, covering almost all parameters where δ is small, indicates technologies for which $\tau = 0$ subsidies lower welfare if $\tau = 0.2$. The red area, mainly consisting of parameters where LBD is limited, indicates technologies where there is either an efficiency gain or the existence of multiple equilibria when $\tau = 0.2$ means that we cannot sign whether welfare is increased or reduced.

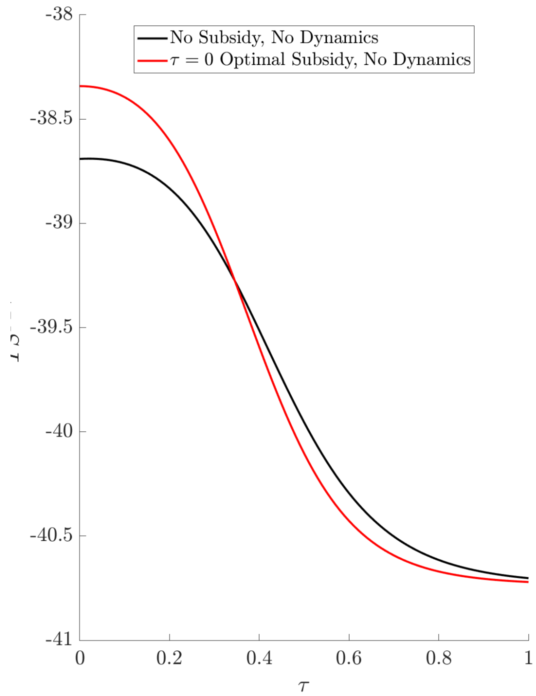
The white area indicates parameters where we cannot solve for $\tau = 0$ optimal subsidies or we cannot solve for the equilibria when $\tau = 0.2$ when these subsidies are in place. The fact that the white areas cover so many parameters may seem surprising. It reflects the extreme values of some of the required subsidies or the extreme values of the equilibrium

Figure E.5: Optimal Subsidy Schemes and Welfare as Functions of τ Assuming Static Seller Behavior for the $M = 3$ and $M = 30$ Examples. Negative subsidies are taxes on the laggard when it makes a sale.

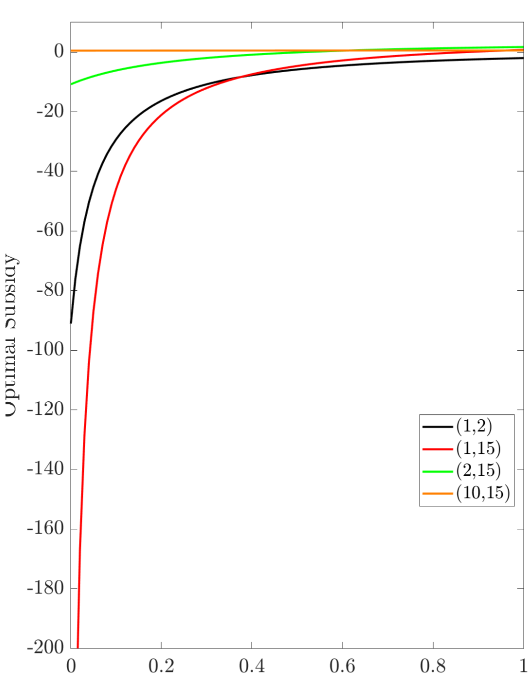
(a) $M = 3$: Optimal Subsidy Levels



(b) $M = 3$: TS^{PDV} with $\tau = 0$ Optimal Subsidies



(c) $M = 30$: Optimal Subsidy Levels (Selected States)



(d) $M = 30$: TS^{PDV} with $\tau = 0$ Optimal Subsidies

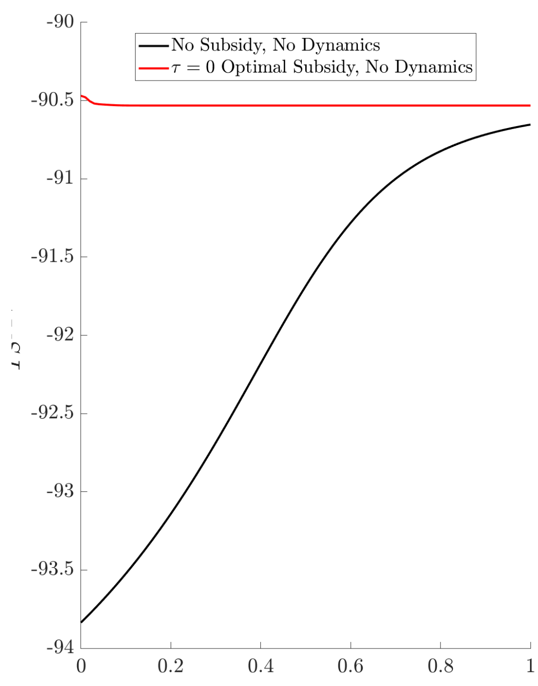
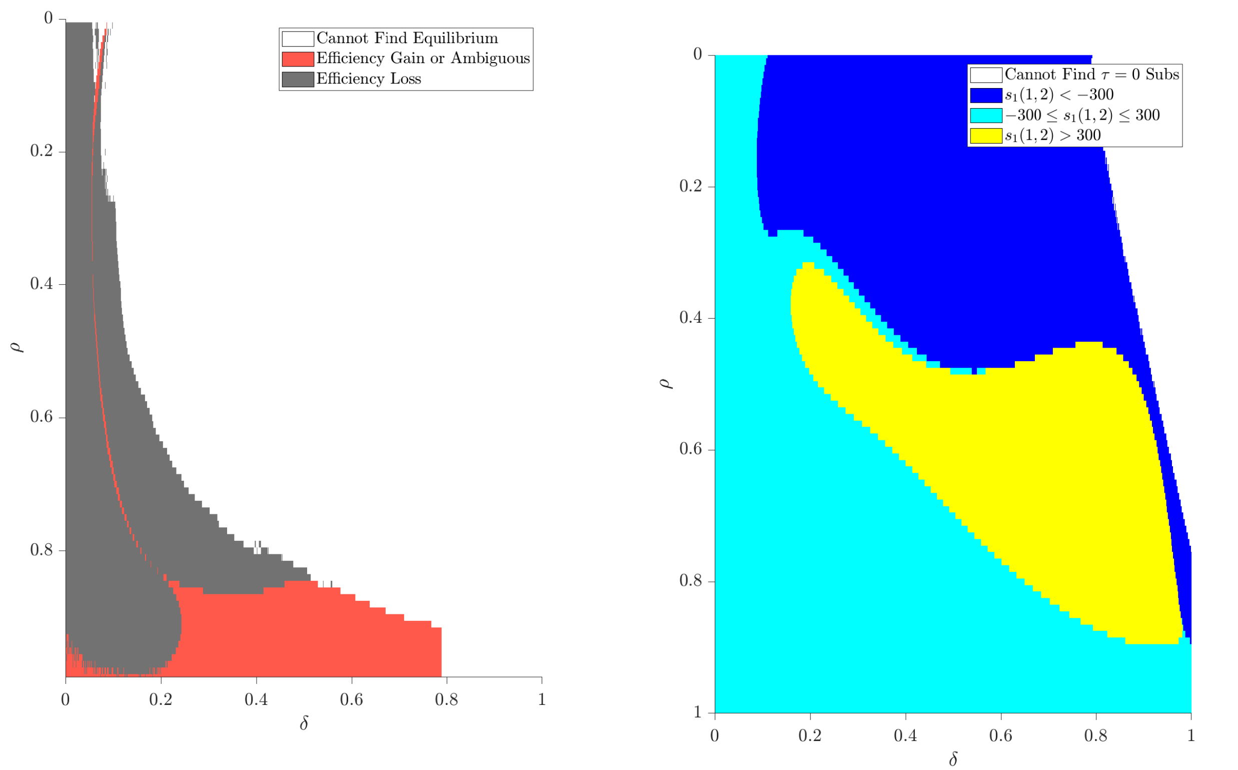


Figure E.6: Optimal $\tau = 0$ Subsidies and Welfare in the $M = 3$ Model. Panel (a) shows, for the values of ρ and δ for which we can identify equilibria, parameters where $\tau = 0$ optimal subsidies increase or decrease welfare when $\tau = 0.2$ (compared to no subsidy equilibria). Panel (b) shows the value of $\tau = 0$ subsidies to the laggard in state (1,2). A negative subsidy is a tax on a laggard sale.

(a) Welfare Effect of $\tau = 0$ Subsidies if $\tau = 0.2$

(b) Level of $\tau = 0$ Laggard Subsidy in State (1,2)



choice probabilities when these subsidies are in place and $\tau \neq 0$. As illustration, panel (b) shows the level of the subsidy the planner would want to give to a laggard making a sale in state (1,2) (negative value indicates a tax). For $\delta > 0.1$, the subsidies or taxes can be extremely large, which is the source of numerical problems. It is also interesting how a small change in ρ can switch the optimal scheme from providing a laggard with a very large subsidy to requiring the laggard to pay a very large tax.

E.5 Alternative Concentration Restriction Policies.

Text Section 5.5 shows the effects of, *inter alia*, a concentration restriction policy, where the leader i has to pay a compliance penalty of $\chi \times \max\{0, D_i - \psi\}$ where $\chi = 50$ and $\psi = 0.75$. As $D_i = 0.5$ when firms are symmetric, and the maximum D_i approaches 1, $\psi = 0.75$ is a natural value to consider. We choose $\chi = 50$ as an example of a policy which lowers concentration but which still provides some probability that a firm will establish a know-how advantage that will lead to the compliance cost being incurred.

Figure E.7 shows how policies with different χs affect HHI^{32}, HHI^{200} and TS^{PDV} (recall that we do not count the compliance cost as decreasing total surplus) for the illustrative technology parameters, as a function of τ. Consistent with what one would expect, increases in χ lower concentration for values of τ where the share threshold would likely be breached with no policy in effect. For the values of χ that we consider, welfare falls as χ increases.[52]

E.6 Contracts for a Repeat Buyer.

We consider three scenarios. Here, we briefly summarize the math involved in each case.

No contracts. In this scenario, there is period-by-period competition for both types of buyer as in our baseline model. However, the repeat buyer is forward-looking. Therefore, the equilibrium can be defined by two vectors of equilibrium prices $p_k^{R*}(\mathbf{e})$ and $p_k^{A*}(\mathbf{e})$, a vector of seller values, $V_1^{S*}(\mathbf{e})$ and a vector of buyer values $V^{R*}(\mathbf{e})$ for the repeat buyer.

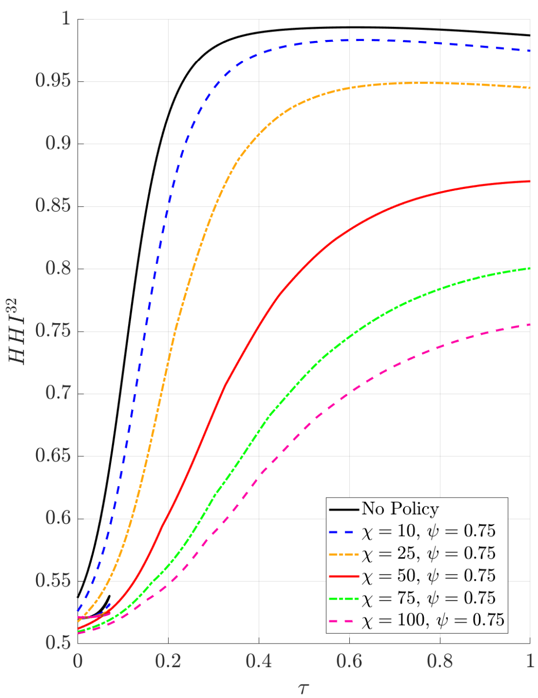
The repeat buyer's values, choice probability and the bargained price first-order conditions when the repeat buyer is the purchaser, are defined as in Appendix E.2, replacing b^p with θ. The seller values can then be defined as

$$\begin{aligned} & V_1^{S*}(\mathbf{e}) \\ =& \theta D_1^{R*}(\mathbf{e})(p_1^{R*}(\mathbf{e}) - c(e_1)) + (1-\theta) D_1^{A*}(\mathbf{e})(p_1^{A*}(\mathbf{e}) - c(e_1)) \\ & + \sum_{k=1,2} [\theta D_k^{R*}(\mathbf{e}) + (1-\theta) D_k^{A*}(\mathbf{e})] \mu_{1,k}^S(\mathbf{e}) \end{aligned} \tag{47}$$

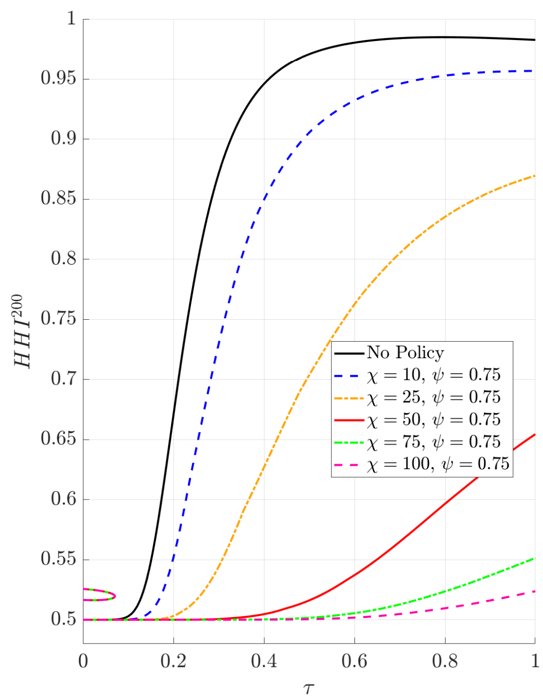
[52]We note, however, that one can find examples where a small χ can increase TS^{PDV}. For example, when $\tau = 0.4$, $TS^{PDV} = -90.4880$ when $\chi = 2$ and -90.4883 when $\chi = 0$ (no policy).

Figure E.7: Concentration and Welfare as a Function of τ for Alternative Compliance Cost Parameters and the Concentration Restriction Policy for the $M = 30$ Example Parameters.

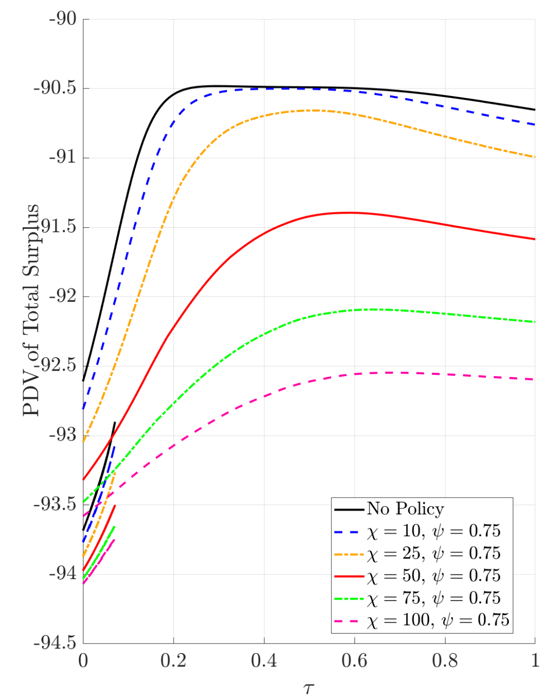
(a) HHI^{32}



(b) HHI^{200}



(c) TS^{PDV}



where θ is the probability of a repeat buyer is drawn.

R **has symmetric fixed price contracts with both sellers.** In this case we assume that, when R is the buyer, the probability that seller k makes the sale is $\phi_k = \frac{1}{2}$. The beginning of period value for firm 1 is given by

$$\begin{aligned}
&V_1^{S*}(\mathbf{e}) \\
=&(1-\theta)D_1^*(\mathbf{e})(p_1^*(\mathbf{e}) - c(e_1)) + \theta\phi_k(p^R - c(e_1)) \\
&+ \sum_{k=1,2} [\theta\phi_k + (1-\theta)D_k^*(\mathbf{e})]\mu_{1,k}^S(\mathbf{e})
\end{aligned} \tag{48}$$

where θ is the probability of a repeat buyer is drawn. p^R denotes an arbitrary contracted fixed price of the repeat buyer. ϕ_k is the purchase probability of the repeat buyer from seller k. $\mu_{1,k}^S(\mathbf{e})$ is seller 1's continuation value when seller k makes the sale. The first-order conditions for the bargained atomistic buyer prices are the same as in the baseline model.

R **has an exclusive fixed price contract with one seller.** The key innovation in this case is that the sellers are no longer symmetric. Without the loss of generality, we assume seller 1 is contracted with R, and that it therefore makes a sale in every period R is chosen. Therefore $\phi_1 = 1$ and $\phi_2 = 0$.

The beginning of period value for firm 1 (V_1^S) is then given by

$$\begin{aligned}
&V_1^{S*}(\mathbf{e}) \\
=&(1-\theta)D_1^*(\mathbf{e})(p_1^*(\mathbf{e}) - c(e_1)) + \theta(p^R - c(e_1)) \\
&+ \sum_{k=1,2} [\theta\phi_k + (1-\theta)D_k^*(\mathbf{e})]\mu_{1,k}^S(\mathbf{e})
\end{aligned} \tag{49}$$

where θ is probability of a repeat buyer is drawn. p^R denotes some arbitrary contracted fix price of the repeat buyer, which is not solved for.

Seller 2's beginning of period value follows

$$V_2^{S*}(\mathbf{e}) = (1-\theta)D_2^*(\mathbf{e})(p_2^*(\mathbf{e}) - c(e_2)) + \sum_{k=1,2} [\theta\phi_k + (1-\theta)D_k^*(\mathbf{e})]\mu_{2,k}^S(\mathbf{e}) \tag{50}$$

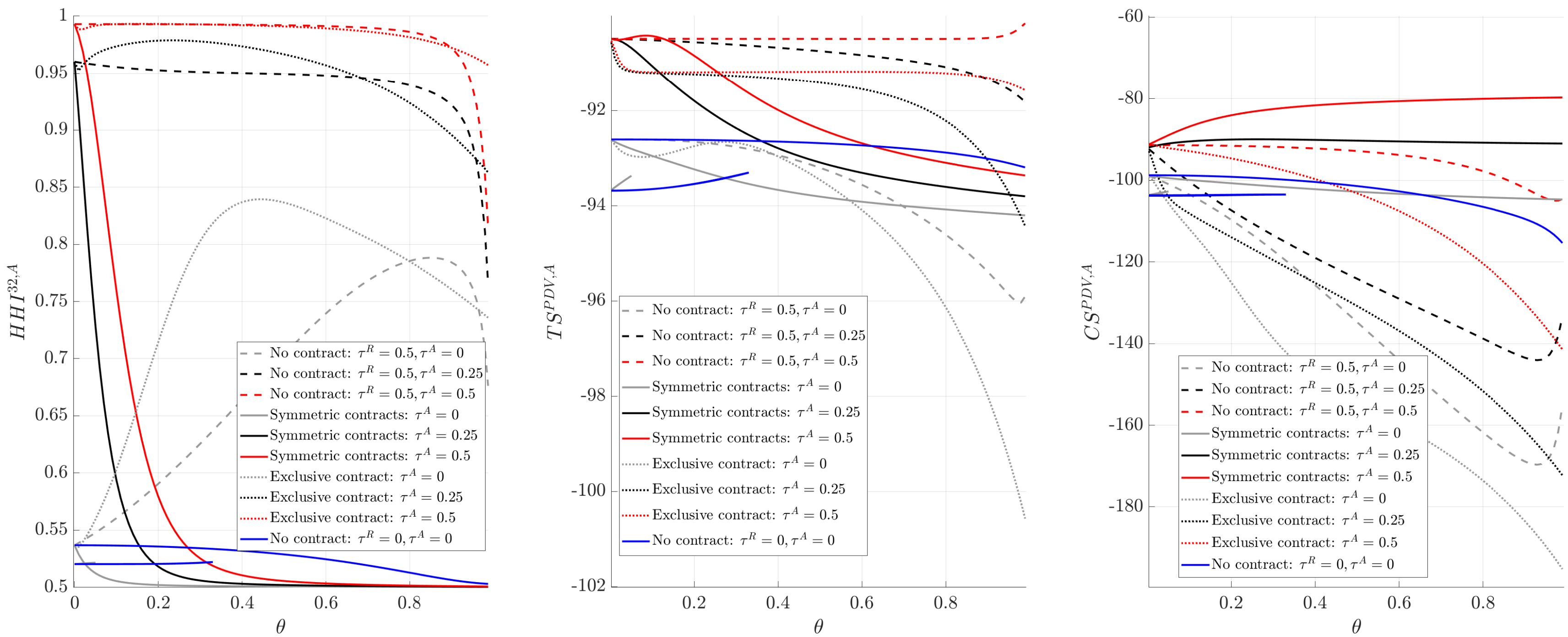
where θ is the probability of a repeat buyer is drawn. $\mu^S_{2,k}(\mathbf{e})$ is seller 2's continuation value when seller k makes the sale.

The first-order conditions for prices to A type buyers have the same structure as in the baseline model, but now there are separate first-order conditions for each seller and they may chage different prices in states where they have the same know-how.

Results. Figure E.8 reports the values of $HHI^{32,A}$, $TS^{PDV,A}$ and $CS^{PDV,A}$ as functions of θ for the values of τ^A that we are considering. These values provide the basis of the summary results reported in Table 1.

Figure E.8: Concentration, Welfare and Buyer Surplus for Sales to Short-Lived Buyers, as a Function θ (Proportion of Sales Made to Repeat Buyers). $M = 30$ model with $\rho = 0.75$ and $\delta = 0.023$. Surplus is calculated as if there is an A-type buyer every period for all θ.

(a) $HHI^{32,A}$ (b) $TS^{PDV,A}$ (c) $CS^{PDV,A}$



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