Combinatorial versus Holistic Procurement: The Role of Information Frictions*

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Abstract

Large-scale infrastructure projects often involve multiple component tasks and require a range of skills. When procuring such projects, an auctioneer can either organize a combinatorial auction—which allows a firm to bid on the entire project or on individual components as independent entities—or require holistic proposals, whereby specialized firms form consortia to pool their expertise and bid jointly. This paper compares the performance of these two auction formats. In the absence of information frictions, a holistic procurement auction outperforms a combinatorial auction and achieves full allocative efficiency at lower procurement prices. However, information frictions lead to non-assortative matching of firms or strategic behavior within consortia, which may cause the holistic procurement to underperform the combinatorial auction in terms of allocative efficiency and procurement price.

Keywords: Procurement Auctions; Complementarity; Joint Bidding; Information Frictions; VCG Auction.

JEL Classification Codes: D44, D82, H57.

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1 Introduction

Large-scale infrastructure projects, such as the construction of railways, bridges, expressways, and public utility facilities, require complex engineering solutions and multidisciplinary expertise. For instance, constructing an airport demands skills ranging from building terminals and runways to installing electrical and telecommunication networks. This compels contractors with complementary skills to form alliances or consortia, which enables them to pool their expertise and jointly bid for contracts. For example, the Grupo Unidos por el Canal consortium, composed of companies from Spain, Italy, and Belgium, was awarded the contract for the Panama Canal expansion due to their combined capabilities. Estache and Iimi [1] study 221 public contracts for road, water and sewage, and electricity projects in 29 developing countries, and find that consortia account for about 25 to 30 percent of total bidders in procurement auctions for infrastructure projects.

A similar practice can be observed in the execution of complex tech projects that require integrating expertise in multiple areas. The EU's Single European Sky ATM Research (SESAR) program, aimed at modernizing air traffic management (ATM) across Europe, is executed by the SESAR Joint Undertaking (SESAR JU), a public-private partnership that consists of several leading players in the aviation industry, such as Thales (the ATM system) and Frequentis (communication systems). In developing the Federal Health Insurance Exchange website, CGI Federal, QSSI, and Terremark, along with several other firms, formed a consortium, with each handling a specific component.

Given the multifaceted nature of large projects, procurement agencies face critical decisions in designing auction rules. They may require holistic proposals that integrate all aspects of the project, as seen in the Dutch government's procurement for its offshore wind farm projects, which allows only bidding entities that are capable of managing the project's full life cycle. The contract for one of the projects, Hollandse Kust Noord wind farm, was awarded to a joint venture formed by Shell and Eneco. The holistic approach would thus force specialized contractors to form consortia. Conversely, agencies might adopt a combinatorial auction format, which allows firms to bid on either the entire project or individual components as independent entities without joining consortia.[1] This method can be exemplified by the Doha Metro project in Qatar, which permitted bids for specific tasks such as tunneling, station construction, or rail systems. A similar strategy was adopted by the

[1] Combinatorial auctions, which allow bids for bundles or combinations of items, are helpful in addressing complementarity issues in various situations, such as the allocation of electromagnetic spectra (see McMillan [2]; Cramton [3]; Ausubel and Baranov [4]; and Hafalir and Yektaş [5]; among many others); school meals auctions (Olivares, Weintraub, Epstein, and Yung [6]); and airport time slots [7]. See De Vries and Vohra [8] and Elmaghraby and Keskinocak [9] for surveys.

I-595 Express Corridor Improvements Project, which upgraded a major highway corridor in Florida, and the Sydney Light Rail Project. When the U.S. Department of Defense (DoD) procured its Joint Enterprise Defense Infrastructure (JEDI) Cloud service, the procurement mechanism allowed for multiple winners on separate components, although the project was ultimately awarded to a single bidder, Microsoft Azure.

These mixed observations raise important questions. For policymakers, which auction format could ensure allocative efficiency—i.e., awarding the contract to the most efficient firms? For procurement agencies, which auction format could generate lower procurement costs? This paper seeks to answer the above questions by comparing the performance of combinatorial auctions, which allow for specialized firms to bid on components, with holistic procurement auctions, which force joint bidding.

Joint bidding by competing firms has raised significant anti-competitive concerns and triggered extensive regulatory oversight. In contrast, bidding consortia formed by *complementary* firms are viewed more favorably because such arrangements presumably facilitate collaboration.[2,3] However, our analysis suggests caution when it comes to joint bidding, even by complementary firms. The choice between combinatorial auctions and holistic procurement auctions crucially depends on the prevailing information environment and can be subject to information frictions between the parties involved (see also Davis, Hu, Hyndman, and Qi [11]).

We consider the procurement of a project that involves two component tasks, A and B.[4] While some firms are able to execute the entire project independently, others specialize in either task A or task B. We refer to the former as type AB firms and the latter—i.e., those specialized in task $\alpha \in \{A, B\}$—as type α. If the auctioneer enforces a holistic approach, a type A firm must form a consortium with a type B partner to bid jointly.[5] The auctioneer may also organize a combinatorial auction and allow all firms to submit independent bids, such that type AB firms bid on the entire project and type α firms bid on their respective specialized components. The auction mechanism decides whether to assign the entire project

[2]In fact, although the Energy Policy and Conservation Act enacted in 1975 and the Outer Continental Shelf Lands Act Amendment of 1978 ban joint bidding by major oil companies for outer continental shelf (OCS) leases, the authority reserves the right to allow joint bidding by said companies on lands that have extremely high-cost exploration or development problems and on lands where exploration and development will not occur unless exemptions are granted (Millsaps and Ott [10]).

[3]In 2018, the Italian Competition Authority (ICA) approved the joint bidding agreement for the contract manufacturing of plasma therapeutic products derived from blood donations between Grifols and Kedrion on the grounds that the two barely compete with each other in the relevant market.

[4]Our results remain qualitatively robust if multiple tasks are involved.

[5]We prioritize allocative efficiency as an outcome measure of the procurement and thus assume second-price auctions when joint bidding is allowed, since first-price auctions are known to be inefficient with asymmetric bidders.

to a type AB firm or award component-specific contracts to specialized firms separately. We assume that the auctioneer values both allocative efficiency—which requires that the project be allocated to firms with the lowest completion costs—and procurement price. We then compare the performance of the two auction formats in these respects.

Our study highlights the vital role of information frictions in choosing the proper approach to procuring projects with multiple components. With holistic procurement, information frictions could emerge when specialized firms bid jointly. First, information friction could arise in the matching process when multiple type A firms and type B firms form consortia: Without knowing the private costs of potential partners, firms with lower costs may team up with complementary firms with high costs, and thus cause an inefficient matching outcome and inflate the procurement price. Second, information friction can also arise between matched firms: The firms within a consortium may not know each other's private cost and thus behave *strategically* instead of seeking to maximize joint profit, which could also cause inefficiency and elevate the procurement price.

We begin with a base case that assumes away information friction. As a result, firms are assortatively matched; those within each consortium share their cost information, so they place a joint bid to maximize total profit. In this case, the holistic procurement auction delivers superior performance over the combinatorial auction: The former approach achieves full allocative efficiency at a lower expected procurement price (Proposition 1).

However, information friction complicates the comparison. We consider two cases, each of which depicts one of the two types of information frictions described above. The first examines the role played by the information friction entailed by the matching process—such that type A and type B firms are matched randomly—while assuming that cost information is shared within each consortium. In this case, joint bidding undermines allocative efficiency. Further, the auctioneer may also suffer a higher procurement price on average: When the numbers of specialized firms are sufficiently large, the combinatorial auction outperforms the holistic procurement auction in terms of the expected procurement price (Proposition 2).

We then move on to explore the case of information friction within consortia. To isolate its role, we abstract away the friction embedded in the matching process and focus on a setting with only one type A firm, one type B firm, and one type AB firm, which corresponds to the "local-local-global (LLG)" model in the auction literature (see, e.g., Krishna and Rosenthal [12]; Ausubel and Baranov [13]).[6] In this case, the type A and the type B firms in the consortium do not share cost information and act strategically to maximize their own profits. The scenario is equivalent to a case of subcontracting, with one firm the prime contractor and the other the subcontractor. The former proposes a mechanism to elicit private information

[6]Type A and type B firms are viewed as local bidders, and the type AB firm a global bidder.

from the latter and offers a transfer payment. The usual *double marginalization* problem arises because the parties within the consortium maximize their own profits sequentially (Proposition 3). As a result, the holistic procurement auction causes inefficiency and could lead to higher procurement prices than the combinatorial auction (Proposition 4) due to firms' strategic behavior under information friction.

Our results not only provide theoretical insights but also generate useful implications for policy and practice, which we discuss in the Concluding Remarks.

Relation to the Literature This paper contributes to the broad and extensive literature in economics and operations management that explores the fundamentals of procurement mechanisms. Since the seminal work of Vickrey [14], a wealth of scholarly effort has been devoted to characterizing the procurement mechanisms that achieve allocative efficiency (Green and Laffont [15]; Krishna and Perry [16]; Chen et al. [17]) or minimize procurement costs (Myerson [18]; McAfee and McMillan [19]; Wan et al. [20]; Gujar and Narahari [21]; Beil et al. [22]; Choi et al. [23]).

In particular, our paper is more closely related to the burgeoning literature that evaluates alternative ways to organize the procurement of multiple components or, equivalently, the outsourcing of interdependent services (Li, Sun, Yan, and Yu [24]; Chen and Li [25]; Davis, Hu, Hyndman, and Qi [11]). These studies compare bundled procurement with independent or sequential auctions for components and typically focus on auction mechanisms. Hu and Wang [26] and Chen, Mihm, and Schlapp [27] consider contest mechanisms for the procurement of products with multiple components or attributes and compare a single contest for the entire system or two independent contests for respective components. We adopt auction mechanisms to model the procurement of large projects. In contrast to these studies, we explore a situation in which the component tasks involved in a project must be procured together;[7] instead of allowing for separate auctions for components, we compare holistic procurement (i.e., procuring a bundle) with combinatorial procurement, in which all bidders participate in a single auction but each can bid on either the bundle or a single component.

This paper is naturally linked to studies of joint bidding within the auction literature. Previous literature has conventionally examined joint bidding by firms that are able to finish the entire project on their own and would otherwise compete against each other. These studies thus highlight the anti-competitive effects of joint bidding. In contrast, an expansive literature examines scenarios in which joint bidding could catalyze synergy among

[7]This assumption is reasonable when procuring components in separate auctions is either infeasible (e.g., due to excessive administrative hurdles or potential hold-up problems) or not economical (e.g., individually capable firms enjoy significant cost savings by finishing the entire project).

bidders—e.g., information sharing (DeBrock and Smith [28]; Levin [29]; Mares and Shor [30]; Mares and Shor [31]); capital pooling (Hoffman, Marsden, and Saidi [32]); and value creation or ownership-sharing (Marquez and Singh [33])—and explores the tension between the synergy enabled by joint bidding and its anti-competitive effects. Our paper joins this research stream by considering firms that possess complementary skills and could join forces to bid on projects with multiple components. We compare combinatorial auctions—which accommodate specialized firms as independent bidders—with auctions that require holistic proposals and thus require joint bidding by specialized firms. Our results demonstrate that the choice of auction format could depend on the information friction in the environment.

When information friction arises within a consortium, the interaction between the allied firms resembles that in a subcontracting arrangement. Our paper is thus connected to the literature on subcontracting. This strand of the literature has explored a broad array of issues, ranging from how subcontracting affects auction outcomes (Gale, Hausch, and Stegeman [34]; Marion [35]; Jeziorski and Krasnokutskaya [36]) and how the formats of procurement auctions affect entry and subcontracting behavior (Branzoli and Decarolis [37]) to how contract renegotiation or "bid-shopping" affects the performance of auctions in the presence of subcontracting (Bajari and Tadelis [38]; Miller [39]; Deneckere and Quint [40]). Our interest, however, lies in whether an auction that would lead to joint bidding or subcontracting is the preferred approach to organizing a procurement auction when bidders exhibit complementarity.

The rest of the paper is organized as follows. Section 2 presents the primitives of the model. Section 3 analyzes a base case that compares a holistic procurement auction without information friction to a combinatorial Vickrey-Clarke-Groves (VCG) auction. Section 4 analyzes auctions with information friction, and Section 5 concludes. Omitted proofs are provided in the Appendix.

2 Model and Preliminaries

In this section, we lay out the primitives of the model.

2.1 Primitives

An auctioneer seeks to procure a project comprised of two components, A and B. N_A firms are specialized in component A, N_B firms are specialized in component B, and N_{AB} firms can handle both. We assume that there are significant synergies within each type AB firm, such that the cost of completing one task is equal to the cost of completing both. Consequently, a type AB firm is either awarded the entire project or nothing. Alternatively, this can be interpreted as arising from internal organizational frictions that prevent an individually capable firm from subcontracting one component to an external supplier; thus, the firm must perform both components in-house if it undertakes the project at all. This assumption primarily serves to simplify our exposition. Our main findings remain qualitatively robust as long as some degree of synergy exists within each individually capable firm. We further discuss its implications in Remark 1 at the end of Section 3.

We denote the cost of a representative firm (α, i) by c_i^α, where $\alpha \in \{A, B, AB\}$ and $i \in \{1, 2, \ldots, N_\alpha\}$. A firm's cost is privately known, but it is commonly known that c_i^α is distributed according to a cumulative distribution function $F_i^\alpha(\cdot)$ on $[\underline{c}^\alpha, \overline{c}^\alpha]$, with an associated probability density function $f_i^\alpha(\cdot)$. Throughout the paper, we assume that $f_i^\alpha(c) > 0$ for $c \in [\underline{c}^\alpha, \overline{c}^\alpha]$ and is continuous. Moreover, it is natural to assume that $\overline{c}^{AB} > \underline{c}^A + \underline{c}^B$ and $\overline{c}^A + \overline{c}^B > \underline{c}^{AB}$. The first condition ensures that there exist cost realizations under which two specialized firms together can be cheaper than individually capable firms; the second ensures the reverse. Together, these conditions rule out the degenerate case in which either the individually capable firms or the specialized firms are ex ante dominated and thus have no reason to participate.

Let $\boldsymbol{x} := (x_1^A, \ldots, x_{N_A}^A, x_1^B, \ldots, x_{N_B}^B, x_1^{AB}, \ldots, x_{N_{AB}}^{AB})$ denote the allocation outcome, where $x_i^\alpha \in \{0, 1\}$ indicates whether a firm (α, i) is selected to deliver task $\alpha \in \{A, B, AB\}$. The set of all feasible allocations is given by $X := X^{\text{sep}} \cup X^{\text{ind}}$, where

$$X^{\text{sep}} = \left\{ \boldsymbol{x} \in \{0,1\}^{N_A+N_B+N_{AB}} : \sum_i x_i^A = 1,\ \sum_j x_j^B = 1,\ \sum_k x_k^{AB} = 0 \right\}, \text{ and}$$

$$X^{\text{ind}} = \left\{ \boldsymbol{x} \in \{0,1\}^{N_A+N_B+N_{AB}} : \sum_k x_k^{AB} = 1,\ \sum_i x_i^A = 0,\ \sum_j x_j^B = 0 \right\}.$$

The efficiency of a given allocation $\boldsymbol{x} \in X$ is measured by the actual cost of finishing the project, $\boldsymbol{c} \cdot \boldsymbol{x}$, where $\boldsymbol{c} := (c_1^A, \ldots, c_{N_A}^A, c_1^B, \ldots, c_{N_B}^B, c_1^{AB}, \ldots, c_{N_{AB}}^{AB})$.

The auctioneer could organize a combinatorial auction, such that a type A or type B firm can bid independently on a specific component of the project. Alternatively, the auction can only accept holistic solutions, such that specialized contractors have to form consortia to bid jointly, which we refer to as a holistic procurement auction. We model the former as a VCG auction; the latter is depicted as a second-price sealed-bid auction (SPA). Note that a second-price auction is a special case of a VCG auction for a single item, which ensures the consistency of our comparison across two auction formats.[8] The two formats are then compared in terms of allocative efficiency and expected procurement price, which are considered to be the primary performance measures for the auctioneer.

2.2 Combinatorial Auction: VCG Framework

We adopt the VCG framework to model the combinatorial auction. A VCG auction always awards the project to firms with the lowest completion costs, so it can be viewed as a natural candidate for the choice of project allocation mechanism.

The VCG rule in our context is specified as follows. Given a bid profile $\widehat{\boldsymbol{c}} := (\widehat{c}_1^A, \ldots, \widehat{c}_{N_A}^A, \widehat{c}_1^B, \ldots, \widehat{c}_{N_B}^B, \widehat{c}_1^{AB}, \ldots, \widehat{c}_{N_{AB}}^{AB})$, the resultant allocation $\boldsymbol{x}^*$ is given by $\boldsymbol{x}^* = \arg\min_{\boldsymbol{x} \in X} \widehat{\boldsymbol{c}} \cdot \boldsymbol{x}$,[9] and the payment bidder (α, i) receives is the positive externality it imposes on other firms, which amounts to $\min_{\boldsymbol{x}_{-(\alpha,i)}} \widehat{\boldsymbol{c}}_{-(\alpha,i)} \cdot \boldsymbol{x}_{-(\alpha,i)} - \widehat{\boldsymbol{c}}_{-(\alpha,i)} \cdot \boldsymbol{x}^*_{-(\alpha,i)}$. It is well known that truthful bidding is a weakly dominant strategy in the VCG auction. We focus on this equilibrium throughout.

By setting identity-specific reserve prices for bidders, one can construct a broad class of VCG pricing rules, all of which lead to efficient allocation (Krishna and Perry [16]). We focus on the present setting for computational tractability. It is also worth noting that this mechanism is prior-free, in the sense that its implementation does not require any information about the bidders' cost distribution.

3 Base Case: Holistic Procurement Auction without Information Friction

We assume that each firm's cost is independently distributed and privately known, which requires additional specifications on the matching process and information structure when firms bid as consortia. As discussed previously, two types of information frictions could arise. First, information friction could emerge when specialized firms are matched into consortia. Second, the firms within a consortium may not share their private cost information.

In this part, we focus on the simple scenario without information friction. We then compare the performance of the holistic procurement auction with that of the combinatorial auction. This serves as a base case for our subsequent analysis of joint bidding with information friction. The following defines the associated matching and information structure.

[8] This modeling choice is also motivated by tractability. As is well known, alternative mechanisms such as first-price auctions are difficult to analyze in our setting due to bidders' asymmetry.

[9] If there are ties (i.e., multiple minimizers of $\widehat{\boldsymbol{c}} \cdot \boldsymbol{x}$), one of the minimizers is selected randomly.

Assumption 1. ***(Frictionless Joint Bidding)*** *Suppose that (i) type A firms and type B firms are assortatively matched and (ii) when a type A firm and a type B firm form a bidding consortium, they share their private cost information and seek to maximize their joint profit.*

Firms are sorted into consortia in a cost-efficient manner. The firms within a consortium know each other's cost, and each consortium behaves as an integrated contender for the entire project. We model this procurement mechanism as a second-price sealed-bid auction (SPA). Focusing on the truthful bidding equilibrium, we obtain the following.

Proposition 1. *Under Assumption 1, the holistic procurement auction is efficient and always leads to a strictly lower expected procurement price than the combinatorial auction.*

Proposition 1 states that without information friction, the holistic procurement auction outperforms the combinatorial auction: It ensures allocative efficiency while leading to a lower procurement cost for the auctioneer. This is because, under frictionless joint bidding, a type A firm and a type B firm form a consortium that behaves as a single bidder. This reduces the number of incentive-compatibility constraints relative to separate bidding.

To see the intuition, it is useful to examine a simple example with complete information. Suppose that the auction involves one firm of each type, with $N_A = N_B = N_{AB} = 1$, $c_1^A = c_1^B = 1$, and $c_1^{AB} = 3$. The combinatorial auction—a VCG auction in our context—would allocate the project to the two specialized firms; the payment to each firm equals the externality it imposes on other firms—i.e., the difference in the costs borne by other firms without and with the firm's presence. Note that, in the combinatorial VCG auction, the auctioneer compensates the specialized firms *twice* (to ensure incentive compatibility for *both* firms to report truthfully). Specifically, firm $(A, 1)$ receives a payment of $3 - 1 = 2$: Without firm $(A, 1)$, firm $(AB, 1)$ would complete the project at a cost of 3; with firm $(A, 1)$, firm $(B, 1)$ incurs a cost of 1. Similarly, firm $(B, 1)$ also receives a payment of 2. The total procurement price therefore amounts to 4. In contrast, when the holistic second-price auction is viewed as a form of a VCG mechanism for buying a single item, the auctioneer only compensates the externality of the consortium *once*. In this example, the externality imposed by the consortium is 3, so the procurement price is 3. Proposition 1 generalizes this insight and verifies that the externality imposed by merged firms is lower than the sum of externalities imposed by independent specialized firms.

Remark 1. When some (but not necessarily full) synergies exist within each individually capable firm, holistic procurement with frictionless joint bidding may still lead to lower procurement prices for the same reason as in the above example. To see this, we extend the example and suppose that the type AB firm can also complete project A at cost a and project B at cost b, where $a + b > c_1^{AB} = 3$ captures synergies in joint completion. In the holistic procurement, the allocation and price remain the same as before: The consortium wins the project at a price of 3. In the case in which $(A, 1)$ and $(B, 1)$ win in the VCG auction (i.e., $a > 1$ and $b > 1$) and must be compensated as separate entities, the price in the VCG auction is $\min\{a, 2\} + \min\{b, 2\}$, again higher than that in the holistic auction. While this insight remains valid, holistic procurement—even under frictionless joint bidding—does not always yield lower prices than the VCG auction (or the efficient allocation), because it may prevent cross-matching among internal units of individually capable firms.[10] This resonates with our subsequent analysis, which demonstrates that matching frictions among specialized firms may cause the combinatorial approach to outperform the holistic approach.

4 Analysis: Joint Bidding with Information Friction

We now examine scenarios with information frictions in place. As previously stated, two types of information frictions could arise: One concerns the matching process and the other the coordination between specialized firms within a consortium. Sections 4.1 and 4.2 each focus on one type of information friction to highlight its role. The analysis shows that the holistic procurement auction that adopts SPA rules is no longer efficient in the presence of either type of information friction and could result in higher expected procurement prices than the combinatorial auction.

4.1 Information Friction in the Matching Process

We now consider information friction embedded in the matching process. That is, specialized firms do not have enough information about their potential partners, and each is randomly matched to a partner. We maintain part (ii) of Assumption 1 and assume that firms within a consortium share cost information and cooperate frictionlessly upon the formation of the alliance.

To simplify the analysis, we assume that there are an equal number N of type A and type B firms. Further, by random matching we mean that each type A firm is paired with a type B firm uniformly at random to form a bidding consortium, independent of their cost realizations. The following result then obtains.

[10] For a simple example, consider three type AB firms. Firm 1 can do task A for 5, task B for 7, or the whole project for 11. Firm 2 can do task A for 7, task B for 5, or the whole project for 11. Firm 3 can do task A, task B, or the whole project for 9. Then in the VCG auction, because firms 1 and 2 can submit bids for component tasks, the procurement price is 10; but in the holistic auction, if individually capable firms cannot cross-match to form consortia, the procurement price is 11.

Proposition 2. *Suppose that $N_A = N_B = N \geq 2$, $N_{AB} \geq 0$, and that type A firms and type B firms are randomly matched to bid jointly for the project when the auction requires holistic proposals. The following statements hold:*

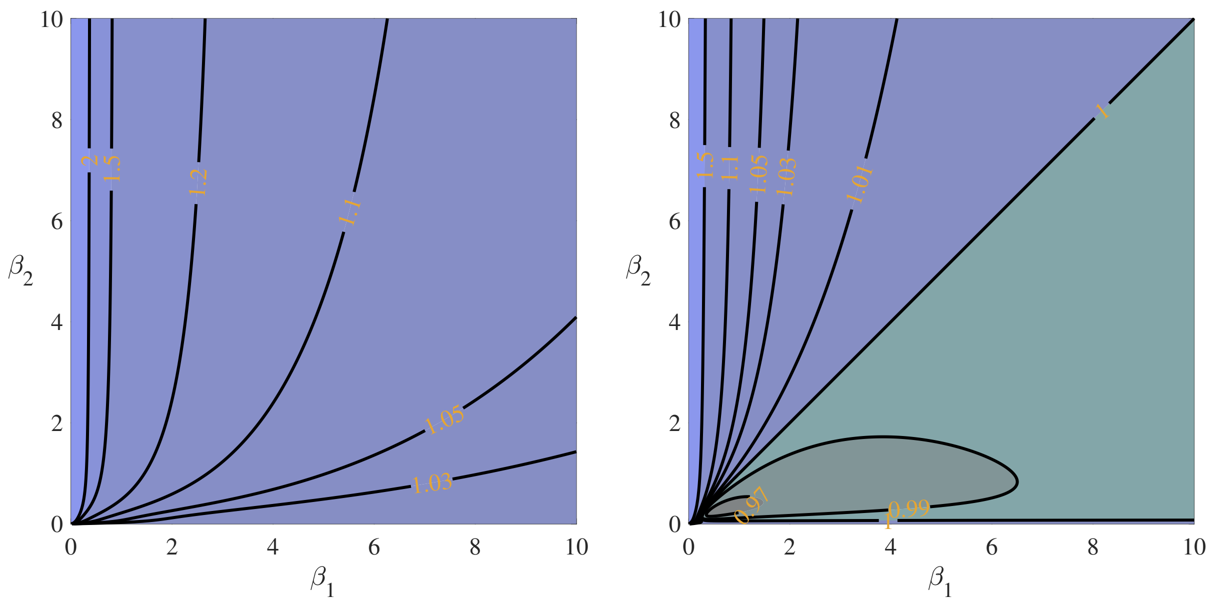
(i) The holistic procurement auction is no longer efficient.

(ii) Suppose further that firms within each type $\alpha \in \{A, B\}$ have the same cost distribution. There exists $\underline{N} \in \mathbb{N}_+$ such that the expected procurement price in the holistic procurement auction is strictly higher than that in the combinatorial auction for all $N \geq \underline{N}$.

Proposition 2(i) is intuitive. Cost-efficient firms could be matched to incompetent partners, which jeopardizes the allocative efficiency of the auction. Obviously, the least costly type A firm may not be matched to the least costly type B firm. The inefficient matching could elevate bidders' costs and soften the competition, and thereby inflate the procurement price. Proposition 2(ii) contends that the expected procurement price of the holistic procurement auction exceeds that of the combinatorial auction when the numbers of specialized firms, N, are sufficiently large. To put this intuitively, the more firms to be matched, the more significant the inefficiency incurred in the matching process and the more severe the distortion to the competition. In the proof, we show that for large N, the procurement price in the VCG auction is essentially the sum of the second-lowest cost among type A firms and the second-lowest cost among type B firms, which converges to $\underline{c}^A + \underline{c}^B$ at the standard order-statistics rate. Under the holistic approach with random matching, the price is higher than the lowest-cost among all consortia—that is, the minimum of N i.i.d. sums $c^A + c^B$. Because achieving a very low sum requires both components to be low *within the same pair*, the minimum consortium cost approaches $\underline{c}^A + \underline{c}^B$ at a much slower rate. This difference in convergence rates underpins the asymptotic price ranking.

Proposition 2 is established in a limiting case as N approaches infinity. It remains to examine whether the result continues to hold for small N. To this end, we present a numerical example. We conduct a comprehensive search over a class of distributions and show that the result often holds even for small N. Moreover, for distributions under which the holistic approach yields a lower expected procurement price, the expected procurement price under the VCG auction is only slightly higher. This numerical exercise also sheds light on which distributions are more likely to make the holistic approach deliver a higher expected procurement price than the VCG auction. We summarize the observations as follows.

Example 1. Suppose that $N_A = N_B = N = 3$ and $N_{AB} = 0$. The private costs, c_i^α, with $\alpha \in \{A, B\}$ and $i \in \{1, \ldots, N\}$ are independent and identically distributed on $[0, 1]$



(a) EC^{SPA}/EC^{VCG}. (b) EP^{SPA}/EP^{VCG}.

Figure 1: Ratios of expected completion costs and expected procurement prices under the holistic procurement auction (SPA) relative to the VCG auction in Example 1.

according to a beta distribution with parameters (β_1, β_2). Further, suppose that in the holistic procurement auction, type A firms and type B firms are randomly matched to form N bidding consortia.

The beta family of distributions is notably representative: By varying (β_1, β_2), it can generate a wide range of shapes, including increasing and decreasing densities, U-shaped densities, and unimodal densities with varying skewness. It also nests several commonly used special cases, including uniform distributions (when $\beta_1 = \beta_2 = 1$), power distributions (when $(\beta_1, \beta_2) = (\beta, 1)$), and arcsine distribution (when $\beta_1 = \beta_2 = \frac{1}{2}$).

On the (β_1, β_2) plane, the contour plots in Figure 1 report ratios of (i) expected completion costs, EC^{SPA}/EC^{VCG}, and (ii) expected procurement prices, EP^{SPA}/EP^{VCG}, under the holistic procurement auction (SPA) relative to those under the VCG auction when $N = 3$.

Figure 1(a) illustrates the efficiency loss in the SPA caused by random matching. From Figure 1(b), we see that even for $N = 3$, the combinatorial approach outperforms the holistic approach for beta distributions with $\beta_1 < \beta_2$, which corresponds to the case of right-skewness (i.e., more mass near 0 on $[0, 1]$). For distributions under which holistic procurement yields lower expected prices, the expected price is typically still above 99% of the expected VCG price and never below 95%.

To see why left-skewness (i.e., probability mass shifted toward high costs) may cause the holistic auction to generate a lower expected price despite matching inefficiencies, consider the following example. Assume that realized costs are always 0.1, 0.7, and 0.8 for the three type A firms and also for the three type B firms. The realized high costs capture the case of left-skewness. The VCG price is then always 1.4. However, under holistic procurement and uniformly random matching, it is likely that there are two (low, high) cost pairs—for example, $(0.1, 0.7)$ and $(0.8, 0.1)$—which lead to a relatively low price in the SPA.[11]

4.2 Information Friction within a Bidding Consortium

We now examine information friction within a bidding consortium: Its members do not share cost information and collaborate in an incentive-compatible way instead of seeking to maximize joint profit. To highlight the role played by such information friction, we abstract away the matching process by focusing on a simple setting with $N_A = N_B = N_{AB} = 1$ for the moment, which corresponds to the LLG model in the literature.

In the holistic procurement auction, the costs of firms $(A, 1)$ and $(B, 1)$ remain privately known after they form a consortium, and each firm seeks to maximize its own profit. We assume that the collaboration takes the form of a subcontracting arrangement.[12] Without loss of generality, we let firm $(A, 1)$ be the prime contractor and responsible for designing a subcontracting mechanism to elicit the cost information from the subcontractor, firm $(B, 1)$; firm $(A, 1)$ then comes up with a bid on behalf of the consortium. We refer to this case as *strategic* joint bidding to distinguish it from the case of frictionless joint bidding, as described by Assumption 1(ii).

More formally, the timing of the auction game is as follows. Prior to the auction, the prime contractor announces its subcontracting mechanism, which consists of a bidding rule $\widehat{c}(c_1^A, c_1^B)$ and a transfer rule $t(c_1^A, c_1^B)$. The prime contractor also reveals its own cost type c_1^A to the subcontractor.[13] Thus the subcontractor knows that the subcontracting scheme, conditional on the prime contractor's type, is $(\widehat{c}(c_1^A, \cdot), t(c_1^A, \cdot))$. Then the subcontractor reports its cost to the prime contractor. Given the reported cost $\widehat{c}_1^B$, the prime contractor

[11]More precisely, the probability that there exist two (low, high) cost pairs is 2/3, which yield a price of either 0.8 or 0.9. With the remaining probability 1/3, there is one (low, low) pair and two (high, high) pairs, which yield a procurement price similar to that under the VCG auction.

[12]Other cost-sharing protocols are possible in theory. We focus on this particular mechanism because it is widely used in practice. Moreover, it delivers a key qualitative insight that we expect to hold more generally: Under any cost-sharing arrangement in which both firms participate voluntarily and in an incentive-compatible manner, the consortium's bid will be (weakly) higher than the sum of true costs. This bid inflation is the key force driving our main result.

[13]Because the prime contractor is risk-neutral and its cost type is not correlated with the subcontractor's, it is without loss of generality to let the prime contractor reveal its type truthfully when announcing the mechanism (see Proposition 11 in Maskin and Tirole [41]).

submits a bid of $\widehat{c}(c_1^A, \widehat{c}_1^B)$ and pays the subcontractor $t(c_1^A, \widehat{c}_1^B)$.[14] Meanwhile, firm $(AB, 1)$ submits its bid $\widehat{c}_1^{AB}$. Finally, the auction concludes according to the standard second-price rule.

Our analysis adopts the solution concept of principals' equilibrium proposed by Myerson [42]. This equilibrium notion requires that given firm $(AB, 1)$'s bidding strategy, the subcontracting mechanism maximizes the prime contractor's expected profit subject to the subcontractor's incentive compatibility, which ensures its truthful reporting. Also, given the bidding rule in the subcontracting mechanism and the belief that the subcontractor reports truthfully, firm $(AB, 1)$'s bidding strategy maximizes its expected profit.

To simplify the equilibrium analysis, we impose a standard regularity condition on the subcontractor's cost distribution, which is given by Assumption 2.

Assumption 2. ***(Regular Cost Distribution)*** The subcontractor's virtual cost, $\tilde{c}_1^B := c_1^B + \frac{F_1^B(c_1^B)}{f_1^B(c_1^B)}$, is weakly increasing in c_1^B.

It is straightforward to see that bidding truthfully is a weakly dominant strategy for firm $(AB, 1)$ in the SPA: It maximizes firm $(AB, 1)$'s expected profit regardless of the mechanism chosen by the prime contractor. We obtain the following.

Proposition 3. *Suppose that $N_A = N_B = N_{AB} = 1$ and Assumption 2 holds. In the principals' equilibrium of the auction game, the type AB firm bids $\widehat{c}_1^{AB,*}(c_1^{AB}) = c_1^{AB}$ and the type A firm, as the prime contractor, adopts the bidding rule of $\widehat{c}^*(c_1^A, c_1^B) = c_1^A + \tilde{c}_1^B$—where $\tilde{c}_1^B$ is the type B firm's virtual cost—and the transfer rule of $t^*(c_1^A, c_1^B) = \bar{F}_1^{AB}(\widehat{c}^*(c_1^A, c_1^B))c_1^B + \int_{c_1^B}^{\bar{c}_1^B} \bar{F}_1^{AB}(\widehat{c}^*(c_1^A, x))dx$, with $\bar{F}_1^{AB}(\cdot) := 1 - F_1^{AB}(\cdot)$.*

Within the consortium, the prime contractor and subcontractor maximize their respective profits sequentially. This generates a markup structure analogous to double marginalization in a vertical supply chain. In such a supply chain, each successive monopolist adds its own margin on top of upstream costs, so that the final price exceeds that under an integrated monopoly. An analogous phenomenon arises here: The prime contractor must concede an information rent to the subcontractor in exchange for truthful cost reporting, and then passes this rent through to the consortium's bid by bidding $c_1^A + \tilde{c}_1^B$ rather than the true cost $c_1^A + c_1^B$. The information rent collected by the consortium in the SPA is, in turn, based on this already marked-up cost, $c_1^A + \tilde{c}_1^B$. In contrast, under the VCG auction, each specialized firm bids directly to the auctioneer; there is no sequential pass-through of one firm's rent into another's bid. This double marginalization problem may therefore lead to a higher expected

[14] The transfer happens regardless of the result of the auction. This is without loss of generality, since both the prime contractor and the subcontractor are risk-neutral.

procurement price under the holistic approach than under the VCG auction. The following result ensues.

Proposition 4. *Suppose that $N_A = N_B = N_{AB} = 1$ and Assumption 2 holds. Then the following statements hold.*

(i) The holistic procurement auction is inefficient.

(ii) When firm $(AB, 1)$ is very strong and likely to win, the holistic procurement auction leads to a higher expected procurement price than the combinatorial auction. Formally, fix a small $\varepsilon > 0$, for any cost distributions such that $\Pr(c_1^{AB} \leq c_1^A + c_1^B) \geq 1 - \varepsilon$, we have that

$$EP^{SPA} - EP^{VCG} \geq \mathbb{E}\left[\frac{F_1^B(c_1^B)}{f_1^B(c_1^B)}\right] - C\varepsilon,$$

where $C := 2\max\left\{\sup_{c\in[\underline{c}^B,\bar{c}^B]} \frac{F_1^B(c)}{f_1^B(c)}, \bar{c}^{AB} - (\underline{c}^A + \underline{c}^B)\right\}$.

Point (i) of Proposition 4 follows immediately from the fact that the consortium bids above its actual cost. To see the intuition for Point (ii), note that when firm $(AB, 1)$ wins, the procurement price is $c_1^A + c_1^B$ in the combinatorial auction; in contrast, the auctioneer pays $c_1^A + \tilde{c}_1^B$ in the holistic procurement auction.

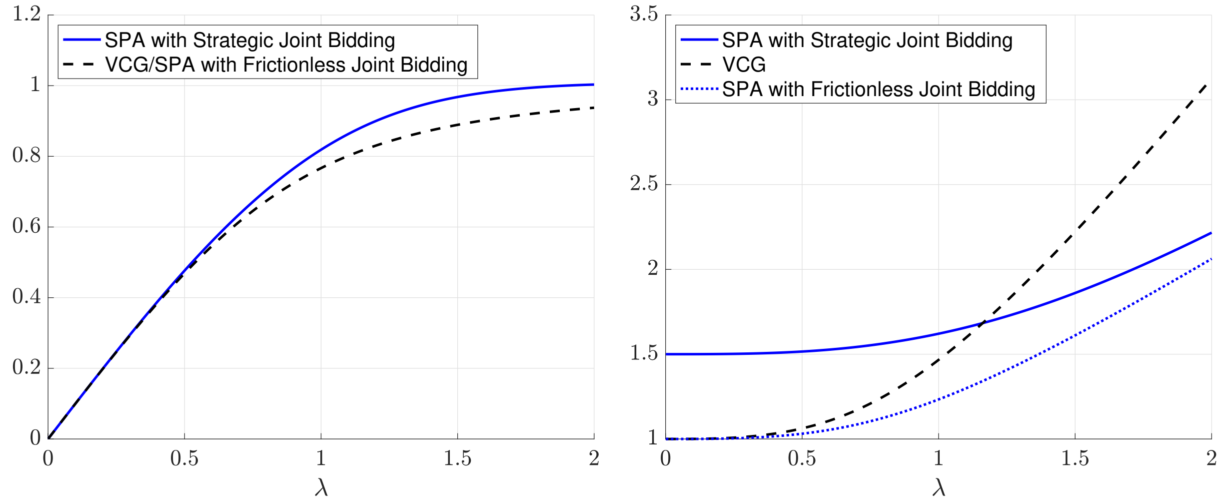
We use a parameterized example to illustrate the results in Proposition 4.

Example 2. Suppose that c_1^A and c_1^B are independent and uniformly distributed on $[0, 1]$. The distribution of c_1^{AB} is the same as that of $\lambda(c_1^A + c_1^B)$, where $\lambda > 0$ captures the relative strength of firm $(AB, 1)$ and the union of firms $(A, 1)$ and $(B, 1)$. Specifically, Firm $(AB, 1)$ becomes stronger as λ decreases.

Figure 2(a) compares the expected completion costs in the holistic procurement auction vis-à-vis those in the base case with frictionless joint bidding and in the combinatorial auction. As predicted by Proposition 4, information friction leads to inefficient allocations.

Figure 2(b) compares expected procurement prices across different cases. By Proposition 4, when firm $(AB, 1)$ is ex ante sufficiently strong—with a small λ in this example—the combinatorial auction outperforms the holistic procurement auction in terms of expected procurement price. The figure demonstrates that this does not require an excessively strong firm $(AB, 1)$ and could emerge even for some $\lambda > 1$, in which case the consortium is ex ante stronger than firm $(AB, 1)$.

Further, we examine the case with type AB firm of moderate strength, i.e., setting λ to 1. Again, we consider a family of beta distributions.



(a) Expected completion costs.

(b) Expected procurement prices.

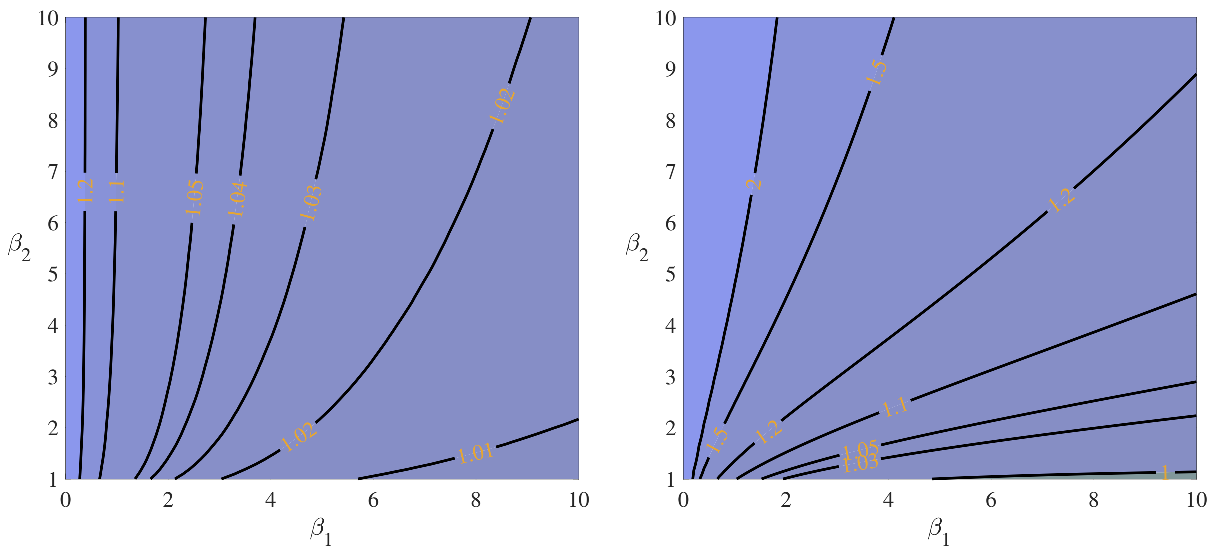
Figure 2: Expected completion costs and procurement prices in the holistic procurement auction and the combinatorial auction in Example 2.

Example 3. Suppose that c_1^A and c_1^B are independent and identically distributed on $[0, 1]$ according to a beta distribution with parameters (β_1, β_2). We assume that $\beta_2 \geq 1$ in accordance with Assumption 2. The distribution of c_1^{AB} is the same as that of $c_1^A + c_1^B$.

Figure 3(a) visualizes the relative efficiency loss under the holistic approach. Figure 3(b) shows that the VCG auction almost always yields a lower expected procurement price, except within a small region in the lower-right corner that resembles a power distribution, $F(c) = c^{\beta_1}$, with large β_1. This observation is consistent with the logic of our result: When β_1 is large, information friction is negligible because the virtual cost, $c\big(1 + \frac{1}{\beta_1}\big)$, is close to the true cost. As established in Proposition 1, in the absence of information friction, the holistic procurement auction outperforms the combinatorial auction. Even within this small region, the expected price under the holistic approach is only marginally lower and remains very close—exceeding 99%—to that under the VCG auction.

Discussion: Multiple Prime Contractors and Subcontractors To underscore the role of information friction within consortia and its resulting double marginalization problem, we abstract away the information friction associated with the matching process and focus on an LLG model that involves a single pair of specialized firms. The underlying logic of our findings can be extended to settings with multiple specialized firms.

Suppose that there are $N_A \geq 1$ type A firms and $N_{AB} \geq 1$ type AB firms. For simplicity and tractability, we assume that a representative type A firm, denoted (A, i), part-



(a) EC^{SPA}/EC^{VCG}.

(b) EP^{SPA}/EP^{VCG}.

Figure 3: Ratios of expected completion costs and expected procurement prices under the holistic procurement auction (SPA) relative to the VCG auction in Example 3.

ners with one of an exclusive set of potential subcontractors, represented by $\{(B,i,k)|k = 1,\dots,N_{Bi}\}$.[15] Let a firm (B,i,k) have a privately known cost c_{ik}^B that is distributed on $[\underline{c}^B, \overline{c}^B]$; further, assume that the virtual cost function $\tilde{c}_{ik}^B := c_{ik}^B + \frac{F_{ik}^B(c_{ik}^B)}{f_{ik}^B(c_{ik}^B)}$ is increasing in c_{ik}^B.

Each prime contractor (A,i) designs a subcontracting mechanism to select one subcontractor and form a joint bid. Specifically, for a given vector of reported costs $(\widehat{c}_{i1}^B,\dots,\widehat{c}_{iN_{Bi}}^B)$, firm (A,i)'s subcontracting mechanism prescribes a joint bid $\widehat{c}_i(c_i^A,\widehat{c}_{i1}^B,\dots,\widehat{c}_{iN_{Bi}}^B)$, a transfer to a potential subcontractor (B,i,k), $t_{ik}(c_i^A,\widehat{c}_{i1}^B,\dots,\widehat{c}_{iN_{Bi}}^B)$, and a probability $q_{ik}(c_i^A,\widehat{c}_{i1}^B,\dots,\widehat{c}_{iN_{Bi}}^B)$ that (B,i,k) will be selected as the partner. As in the LLG case, the prime contractor is assumed to truthfully reveal its cost type to the subcontractors. The principals' equilibrium is characterized as follows.

Proposition 5. *Under Assumption 2, a principal's equilibrium in the holistic procurement auction involves bidding functions* $\widehat{c}_i^{AB,*}(c_i^{AB}) = c_i^{AB}$, $\widehat{c}_i^*(c_i^A, c_{i1}^B,\dots,c_{iN_{Bi}}^B) = c_i^A + \min_{1\le k\le N_{Bi}} \tilde{c}_{ik}^B$*; partner selection rule* $q_{ik}(c_i^A, c_{ik}^B, c_{i,-k}^B) = \mathbb{1}_{\{c_{ik}^B < c_{ik'}^B, \forall k' \neq k\}}$*;*[16] *and transfer rule* $t_{ik}^*(c_i^A, c_{ik}^B, c_{i,-k}^B) = q_{ik}(c_i^A, c_{ik}^B, c_{i,-k}^B) \times \bar{Q}_i\big(\widehat{c}_i^*(c_i^A, c_{ik}^B, c_{i,-k}^B)\big) c_1^B + \int_{c_{ik}^B}^{\bar{c}^B} q_{ik}(c_i^A, \widehat{c}_{ik}^B, c_{i,-k}^B)\bar{Q}_i\big(\widehat{c}_i^*(c_i^A, \widehat{c}_{ik}^B, c_{i,-k}^B)\big) d\widehat{c}_{ik}^B$*, where* $\bar{Q}_i(\widehat{c}) := 1 - Q_i(\widehat{c})$ *and* $Q_i(\widehat{c})$ *is the distribution of the minimum bid of consortium i's opponents, i.e.,* $Q_i(\widehat{c}) := \Pr\big(\min\big\{\min_{1\le j\le N_{AB}} c_j^{AB}, \min_{1\le j\le N_A \text{ and } j\neq i}(c_j^A +$

[15]In the combinatorial auction, we also assume that a type A firm works with only one of its designated subcontractors. This allows us to concentrate on comparing holistic procurement with the combinatorial auction in the absence of matching inefficiencies.

[16]Ties are broken with a fair randomization device.

$\min_{1\leq k\leq N_{Bj}} \tilde{c}^B_{jk})\} \leq \widehat{c}\big)$.

The equilibrium characterization enables a comparison between the holistic procurement auction and the combinatorial auction. Our initial findings are as follows.

Proposition 6. *Suppose that Assumption 2 holds. Further, suppose that $F_i^{AB}(\cdot) = F^{AB}(\cdot)$ and $\underline{c}^A + \underline{c}^B \geq \underline{c}^{AB}$. Then, for sufficiently large N_{AB}, the holistic procurement auction yields a higher expected procurement price than the combinatorial auction.*

Proposition 6 states that with multiple firms, the holistic procurement auction remains more costly to the auctioneer when the number of type AB firms is large. This prediction aligns with our observation in the LLG setting—i.e., Proposition 4—and reinforces the core logic of our analysis. Information friction within each consortium elevates their bids and, in turn, tends to soften competition in a holistic auction. The main idea of the proof is also similar to that of Proposition 4: Under the assumption that $\underline{c}^A + \underline{c}^B \geq \underline{c}^{AB}$, a type AB firm wins almost certainly as N_{AB} becomes sufficiently large.

The following result, which pertains to an alternative extreme case, further clarifies the role of information friction within a consortium.

Proposition 7. *Suppose that $F_{ik}^B(\cdot) = F^B(\cdot)$ and Assumption 2 holds. The holistic procurement auction leads to a lower expected procurement price than the combinatorial auction as N_{Bi} becomes sufficiently large for all i.*

By Proposition 7, the holistic auction may become less costly for the auctioneer when each prime contractor can select a partner from a very large pool of potential subcontractors. Recall that information friction, combined with double marginalization, contributes to the inefficiency of the holistic procurement auction. However, a large N_{Bi} intensifies competition among subcontractors, which reduces their information rent and mitigates double marginalization. This increased competition can thus restore the efficiency of a holistic procurement auction.

5 Concluding Remarks

In this paper, we examine and compare alternative auction approaches to procuring projects that consist of multiple component tasks. The auctioneer can either organize a combinatorial auction that allows specialized firms to bid on component tasks or require holistic proposals and force specialized firms to form consortia and bid on the entire project collectively. Our analysis shows that the holistic procurement auction outperforms the combinatorial auction when information friction is absent: The holistic procurement auction achieves efficient allocation of the project and also yields a lower expected procurement price. However, our further analysis calls for caution with the holistic procurement approach when information friction is present. Information friction could distort the matching of specialized firms and lead competent firms to partner with incompetent peers. It may also arise between firms within a consortium and cause the usual double marginalization problem due to firms' strategic behavior. In either case, the combinatorial auction—which accommodates specialized firms as independent bidding entities—leads to greater allocative efficiency and possibly lower expected procurement prices.

Our analysis demonstrates the key role played by information friction and yields ample implications for procuring complex projects. The choice of auction format could critically depend on the severity of information friction involved in the process of forming consortia and subsequent collaboration within consortia.

Imagine, for instance, a mature market with a relatively small set of established firms that have a long history of interactions. Our results imply that a holistic approach could be a preferred choice given the mild information friction. Conversely, in a market with substantial turnover, more significant information friction would arguably arise due to the lack of interaction between incumbents and entrants. Our results would instead endorse a combinatorial auction format. Furthermore, consider a project that adopts nascent technologies, novel components, or engineering concepts: Information friction is more likely to emerge because of the uncertainty caused by experimentation and learning associated with innovation, which thus calls for a combinatorial approach.

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Appendix Proofs

Proof of Proposition 1

Proof. For a given realization of cost profile $\boldsymbol{c} = (c_1^A, \dots, c_{N_A}^A, c_1^B, \dots, c_{N_B}^B, c_1^{AB}, \dots, c_{N_{AB}}^{AB})$, we assume without loss of generality that $c_1^A \leq \cdots \leq c_{N_A}^A$ and $c_1^B \leq \cdots \leq c_{N_B}^B$. Since costs are drawn from continuous distributions, ties occur with probability zero; we henceforth restrict attention to realizations with strict inequalities $c_1^A < \cdots < c_{N_A}^A$ and $c_1^B < \cdots < c_{N_B}^B$. Because every bidding consortium and every type AB firm bids truthfully in equilibrium, the project is assigned efficiently. That is, firms $(A, 1)$ and $(B, 1)$ win if $c_1^A + c_1^B < c_1^{AB}$ and firm $(AB, 1)$ wins if $c_1^A + c_1^B > c_1^{AB}$.

We first show that the holistic procurement auction leads to a lower procurement price realization by realization. Consider the following three cases:

Case (i): $\boldsymbol{c_1^{AB} \leq c_1^A + c_1^B}$. The procurement price is $\min\{c_1^A + c_1^B, c_2^{AB}\}$ in both the holistic procurement auction and the combinatorial auction.

Case (ii): $\boldsymbol{c_1^{AB} \geq c_2^A + c_2^B}$. The procurement price is $c_2^A + c_2^B$ in both auctions.

Case (iii): $\boldsymbol{c_2^A + c_2^B > c_1^{AB} > c_1^A + c_1^B}$. Firms $(A, 1)$ and $(B, 1)$ win in both auctions. In the combinatorial auction, the payment to firm $(A, 1)$ is $\min\{c_1^{AB} - c_1^B, c_2^A\}$ and the payment to firm $(B, 1)$ is $\min\{c_1^{AB} - c_1^A, c_2^B\}$. So the procurement price is $\min\{c_1^{AB} - c_1^B, c_2^A\} + \min\{c_1^{AB} - c_1^A, c_2^B\}$. In the holistic procurement auction, the payment to the consortium that consists of firms $(A, 1)$ and $(B, 1)$ is c_1^{AB}. Note that

$$
\begin{aligned}
c_1^{AB} - c_1^B + c_1^{AB} - c_1^A &> c_1^{AB}, \\
c_1^{AB} - c_1^B + c_2^B &> c_1^{AB}, \\
c_2^A + c_1^{AB} - c_1^A &> c_1^{AB}, \\
c_2^A + c_2^B &> c_1^{AB}.
\end{aligned}
$$

As a result, $\min\{c_1^{AB} - c_1^B, c_2^A\} + \min\{c_1^{AB} - c_1^A, c_2^B\} > c_1^{AB}$. Therefore, the procurement price is strictly higher in the combinatorial auction than in the holistic procurement auction.

Since Case (iii) occurs with positive probability, the expected procurement price in the holistic procurement auction is strictly lower than that in the combinatorial auction. □

Proof of Proposition 2

Proof. Part (i) of the proposition is obvious, and it remains to prove part (ii). Let $f^\alpha(\cdot)$ and $F^\alpha(\cdot)$ respectively denote the probability distribution and cumulative distribution functions of type α firms.

In the holistic procurement auction, because type A firms and type B firms are randomly matched, each bidding consortium's cost distribution is the same as that of $c_1^A + c_1^B$. This distribution can be described by the support $[\underline{c}^A + \underline{c}^B, \overline{c}^A + \overline{c}^B]$, the probability distribution $f^C(\cdot)$, and cumulative probabilities $F^C(\cdot)$, where the superscript C stands for "Consortium." In particular, for each $x \in [\underline{c}^A + \underline{c}^B, \overline{c}^A + \overline{c}^B]$, it holds that

$$F^C(x) = \int_{u+v\leq x} f^A(u) f^B(v) du dv. \tag{A1}$$

Denote the m-th lowest cost among all joint bidding consortia by $c_{(m)}^C$ and the m-th lowest cost among all type $\alpha \in \{A, B, AB\}$ firms by $c_{(m)}^\alpha$. We show below that for every realization of $c_{(1)}^{AB}$—i.e., the minimum cost among all type AB firms—the limit of the expected procurement price in the holistic procurement auction is higher than that in the combinatorial auction.

Case (i): $\boldsymbol{c_{(1)}^{AB} \leq \underline{c}^A + \underline{c}^B}$. In this case, the type AB firm with the lowest cost wins with certainty in both auctions. (Ties may arise if $c_{(1)}^{AB} = \underline{c}^A + \underline{c}^B$, but they do not matter for the analysis of expected procurement prices.) For every realization of cost profiles, the procurement price in the holistic procurement auction is $\min\{c_{i^\dagger}^A + c_{j^\dagger}^B, c_{(2)}^{AB}\}$, where $i^\dagger$ and $j^\dagger$ are, respectively, the indices of the type A firm and the type B firm that comprise the consortium with the lowest cost. The procurement price in the combinatorial auction is $\min\{c_{(1)}^A + c_{(1)}^B, c_{(2)}^{AB}\}$. Clearly, the procurement price in the combinatorial auction is weakly lower than that in the holistic procurement auction.

Case (ii): $\boldsymbol{c_{(1)}^{AB} > \underline{c}^A + \underline{c}^B}$. Fixing a realization of $c_{(1)}^{AB}$, the expected procurement price in the holistic procurement auction—which we denote by EP^{SPA}—can be bounded from below by

$$\begin{aligned}
EP^{SPA} =& \Pr\left(c_{(2)}^C < c_{(1)}^{AB}\right) \mathbb{E}\left[c_{(2)}^C \,\middle|\, c_{(2)}^C < c_{(1)}^{AB}\right] + \Pr\left(c_{(2)}^C \geq c_{(1)}^{AB}\right) \mathbb{E}\left[\max\{c_{(1)}^C, c_{(1)}^{AB}\} \,\middle|\, c_{(2)}^C \geq c_{(1)}^{AB}\right] \\
\geq& \Pr\left(c_{(2)}^C < c_{(1)}^{AB}\right) \mathbb{E}\left[c_{(1)}^C \,\middle|\, c_{(2)}^C < c_{(1)}^{AB}\right] + \Pr\left(c_{(2)}^C \geq c_{(1)}^{AB}\right) \mathbb{E}\left[c_{(1)}^C \,\middle|\, c_{(2)}^C \geq c_{(1)}^{AB}\right] \\
=& \mathbb{E}\left[c_{(1)}^C \,\middle|\, c_{(1)}^{AB}\right] = \mathbb{E}\left[c_{(1)}^C\right],
\end{aligned}$$

where the inequality follows from the fact that $c^C_{(1)} \leq c^C_{(2)}$ and $c^C_{(1)} \leq \max\{c^C_{(1)}, c^{AB}_{(1)}\}$, and the last equality from the fact that $c^C_{(1)}$ and $c^{AB}_{(1)}$ are independent.

The expected procurement price in the combinatorial auction is lower than $\mathbb{E}\left[c^A_{(2)} + c^B_{(2)}\right]$, since this is the expected procurement price without type AB firms' bids. Therefore, it suffices to show that $\mathbb{E}\left[c^A_{(2)} + c^B_{(2)}\right] < \mathbb{E}\left[c^C_{(1)}\right]$ when N is sufficiently large. Since the absolute level of $\underline{c}^A$ and $\underline{c}^B$ does not matter for this result, for notational ease, we prove it for the $\underline{c}^A = 0$ and $\underline{c}^B = 0$ case.

Carrying out the algebra, we can obtain that

$$\mathbb{E}\left[c^C_{(1)}\right] = \int_0^{\bar{c}^A+\bar{c}^B} \left[1 - F^C(x)\right]^{N-1} dx,$$
$$\mathbb{E}\left[c^\alpha_{(2)}\right] = \int_0^{\bar{c}^\alpha} \left[1 + (N-1)F^\alpha(x)\right]\left[1 - F^\alpha(x)\right]^{N-1} dx,\ \alpha \in \{A, B\}.$$

There exists $\varepsilon > 0$ such that $\frac{f^\alpha(0)}{2} \leq f^\alpha(x) \leq 2f^\alpha(0)$ for each $x \in [0, \varepsilon]$ and $\alpha \in \{A, B\}$. Integration yields that $\frac{f^\alpha(0)x}{2} \leq F^\alpha(x) \leq 2f^\alpha(0)x$. Therefore, for $\alpha \in \{A, B\}$, we have that

$$\begin{aligned}
\mathbb{E}\left[c^\alpha_{(2)}\right] &= \int_0^{\bar{c}^\alpha} \left[1 + (N-1)F^\alpha(x)\right]\left[1 - F^\alpha(x)\right]^{N-1} dx, \\
&= \int_0^{\varepsilon} \left[1 + (N-1)F^\alpha(x)\right]\left[1 - F^\alpha(x)\right]^{N-1} dx \\
&\quad + \int_\varepsilon^{\bar{c}^\alpha} \left[1 + (N-1)F^\alpha(x)\right]\left[1 - F^\alpha(x)\right]^{N-1} dx \\
&\leq \int_0^{\varepsilon} \left[1 + 2(N-1)f^\alpha(0)x\right]\left[1 - \frac{f^\alpha(0)x}{2}\right]^{N-1} dx + N\left[1 - F^\alpha(\varepsilon)\right]^{N-1}\bar{c}^\alpha \\
&= \frac{10}{Nf^\alpha(0)} + o\left(\frac{1}{N}\right).
\end{aligned} \tag{A2}$$

On the other hand, for each $x \in [0, \varepsilon]$, it follows from (A1) that

$$F^C(x) = \int_{u+v\leq x} f^A(u)f^B(v)dudv \leq \int_{u+v\leq x} 4f^A(0)f^B(0)dudv = 2f^A(0)f^B(0)x^2,$$

which in turn implies that

$$\begin{aligned}
\mathbb{E}\left[c^C_{(1)}\right] &= \int_0^{\bar{c}^A+\bar{c}^B} \left[1 - F^C(x)\right]^{N-1} dx \\
&\geq \int_0^{\varepsilon} \left[1 - F^C(x)\right]^{N-1} dx
\end{aligned}$$

$$\geq \int_0^{\varepsilon} \left[1 - 2f^A(0)f^B(0)x^2\right]^{N-1} dx$$
$$= \sqrt{\frac{\pi}{8f^A(0)f^B(0)}} \times \frac{1}{\sqrt{N}} + o\left(\frac{1}{\sqrt{N}}\right). \tag{A3}$$

Combining (A2) and (A3), we can conclude that $\mathbb{E}\left[c^A_{(2)} + c^B_{(2)}\right] < \mathbb{E}\left[c^C_{(1)}\right]$ when N is sufficiently large. □

Proof of Proposition 3

Proof. Given that firm $(AB, 1)$ bids truthfully, the subcontractor's payoff with a report $\widehat{c}^B_1$ is as follows:

$$\pi^B_1(\widehat{c}^B_1; c^B_1) = t\big(c^A_1, \widehat{c}^B_1\big) - \bar{F}^{AB}_1 \left(\widehat{c}\big(c^A_1, \widehat{c}^B_1\big)\right) c^B_1.$$

Incentive compatibility requires that $c^B_1 \in \arg\max_{\widehat{c}^B_1} \pi^B_1(\widehat{c}^B_1; c^B_1)$, which implies that $\frac{\partial \pi^B_1(c^B_1)}{\partial c^B_1} = -\bar{F}^{AB}_1(\widehat{c}(c^A_1, c^B_1))$, where $\pi^B_1(c^B_1) := \pi^B_1(c^B_1; c^B_1)$. It is clear that the prime contractor will set $t(c^A_1, \bar{c}^B) = 0$. Therefore, we have that

$$\pi^B_1(c^B_1) = \int_{c^B_1}^{\bar{c}^B} \bar{F}^{AB}_1 \left(\widehat{c}\big(c^A_1, x\big)\right) dx,$$

which implies that

$$t(c^A_1, c^B_1) = \bar{F}^{AB}_1 \left(\widehat{c}\big(c^A_1, c^B_1\big)\right) c^B_1 + \int_{c^B_1}^{\bar{c}^B} \bar{F}^{AB}_1 \left(\widehat{c}\big(c^A_1, x\big)\right) dx. \tag{A4}$$

Since the prime contractor can set the transfer rule according to (A4) for a given bidding rule to ensure incentive compatibility, we consider its optimal choice of the bidding rule. As is standard in the mechanism design literature, the bidding rule also needs to be increasing in c^B_1 for the incentive compatibility constraint to be satisfied. We ignore the monotonicity constraint for now and show below that it is satisfied in the optimum. The prime contractor's expected payoff, with the expectation taken over c^B_1, is

$$\mathbb{E}_{c^B_1}\left[\int_{\widehat{c}\left(c^A_1, c^B_1\right)}^{\bar{c}^{AB}} \big(x - c^A_1\big) dF^{AB}_1(x) - t\big(c^A_1, c^B_1\big)\right].$$

By plugging (A4) into the expression and changing the order of integration, the prime contractor's expected payoff can be rewritten as

$$\mathbb{E}_{c_1^B}\left[\int_{\widehat{c}(c_1^A,c_1^B)}^{\bar{c}^{AB}} (x - c_1^A - \tilde{c}_1^B)\, dF_1^{AB}(x)\right].$$

It is evident that the above integral is maximized by setting $\widehat{c}(c_1^A, c_1^B) = c_1^A + \tilde{c}_1^B$, which is increasing in c_1^B by Assumption 2. □

Proof of Proposition 4

Proof. Point (i) follows immediately from the fact that the consortium bids above its actual cost. We proceed to prove Point (ii). In the holistic SPA, the procurement price is

$$P^{SPA} = (c_1^A + \tilde{c}_1^B)\, \mathbb{1}_{\{c_1^{AB} \le c_1^A + \tilde{c}_1^B\}} + c_1^{AB}\, \mathbb{1}_{\{c_1^{AB} > c_1^A + \tilde{c}_1^B\}}. \tag{A5}$$

In the combinatorial VCG auction, the procurement price is

$$P^{VCG} = (c_1^A + c_1^B)\, \mathbb{1}_{\{c_1^{AB} \le c_1^A + c_1^B\}} + \big(2c_1^{AB} - (c_1^A + c_1^B)\big)\, \mathbb{1}_{\{c_1^{AB} > c_1^A + c_1^B\}}. \tag{A6}$$

Let $E := \{c_1^{AB} \le c_1^A + c_1^B\}$. Since $\tilde{c}_1^B \ge c_1^B$, on E we also have $c_1^{AB} \le c_1^A + \tilde{c}_1^B$. Therefore, on E firm $(AB, 1)$ wins in both auctions and

$$P^{SPA} - P^{VCG} = (c_1^A + \tilde{c}_1^B) - (c_1^A + c_1^B) = \tilde{c}_1^B - c_1^B = \frac{F_1^B(c_1^B)}{f_1^B(c_1^B)}.$$

Now consider $E^c = \{c_1^{AB} > c_1^A + c_1^B\}$. There are two cases:

Case (a): $c_1^A + c_1^B < c_1^{AB} \le c_1^A + \tilde{c}_1^B$. Then by (A5)–(A6),

$$\begin{aligned} P^{SPA} - P^{VCG} &= (c_1^A + \tilde{c}_1^B) - \big(2c_1^{AB} - (c_1^A + c_1^B)\big) \\ &= (\tilde{c}_1^B - c_1^B) - 2\big(c_1^{AB} - (c_1^A + c_1^B)\big) \\ &\ge -(\tilde{c}_1^B - c_1^B) \\ &\ge -\sup_{c \in [\underline{c}^B, \bar{c}^B]} \frac{F_1^B(c)}{f_1^B(c)}. \end{aligned}$$

Case (b): $c_1^{AB} > c_1^A + \tilde{c}_1^B$. Then by (A5)–(A6),

$$P^{SPA} - P^{VCG} = c_1^{AB} - \big(2c_1^{AB} - (c_1^A + c_1^B)\big) = (c_1^A + c_1^B) - c_1^{AB} \;\ge\; (\underline{c}^A + \underline{c}^B) - \bar{c}^{AB} = -\big(\bar{c}^{AB} - (\underline{c}^A + \underline{c}^B)\big).$$

Therefore, on E^c,

$$P^{SPA} - P^{VCG} \;\geq\; -\max\left\{\sup_{c\in[\underline{c}^B,\overline{c}^B]} \frac{F_1^B(c)}{f_1^B(c)},\; \overline{c}^{AB} - (\underline{c}^A + \underline{c}^B)\right\}. \tag{A7}$$

Taking expectations and decomposing yields:

$$EP^{SPA} - EP^{VCG} = \mathbb{E}\big[(\tilde{c}_1^B - c_1^B)\mathbb{1}_E\big] + \mathbb{E}\big[(P^{SPA} - P^{VCG})\mathbb{1}_{E^c}\big].$$

Using (A7) and $\Pr(E^c) \leq \varepsilon$,

$$EP^{SPA} - EP^{VCG} \;\geq\; \mathbb{E}\big[(\tilde{c}_1^B - c_1^B)\mathbb{1}_E\big] - \max\left\{\sup_{c\in[\underline{c}^B,\overline{c}^B]} \frac{F_1^B(c)}{f_1^B(c)},\; \overline{c}^{AB} - (\underline{c}^A + \underline{c}^B)\right\}\varepsilon.$$

Moreover, since $0 \leq \tilde{c}_1^B - c_1^B \leq \sup_{c\in[\underline{c}^B,\overline{c}^B]} \frac{F_1^B(c)}{f_1^B(c)}$,

$$\mathbb{E}\big[(\tilde{c}_1^B - c_1^B)\mathbb{1}_E\big] = \mathbb{E}[\tilde{c}_1^B - c_1^B] - \mathbb{E}\big[(\tilde{c}_1^B - c_1^B)\mathbb{1}_{E^c}\big] \;\geq\; \mathbb{E}[\tilde{c}_1^B - c_1^B] - \sup_{c\in[\underline{c}^B,\overline{c}^B]} \frac{F_1^B(c)}{f_1^B(c)}\; \Pr(E^c).$$

Using again $\Pr(E^c) \leq \varepsilon$ and combining the two bounds yields

$$\mathbb{E}[P^{SPA} - P^{VCG}] \;\geq\; \mathbb{E}[\tilde{c}_1^B - c_1^B] - 2\max\left\{\sup_{c\in[\underline{c}^B,\overline{c}^B]} \frac{F_1^B(c)}{f_1^B(c)},\; \overline{c}^{AB} - (\underline{c}^A + \underline{c}^B)\right\}\varepsilon,$$

which completes the proof. □

Proof of Proposition 5

Proof. Consider firm (A, i)'s problem while other type A firms and type AB firms play their equilibrium strategies. Given that other potential subcontractors of firm (A, i) report truthfully, firm (B, i, k)'s expected payoff with a report $\widehat{c}_{ik}^B$ is

$$\pi_{ik}^B\left(\widehat{c}_{ik}^B; c_{ik}^B\right) = t_{ik}\left(c_i^A, \widehat{c}_{ik}^B, c_{i,-k}^B\right) - q_{ik}\left(c_i^A, \widehat{c}_{ik}^B, c_{i,-k}^B\right)\bar{Q}_i\left(\widehat{c}\big(c_i^A, \widehat{c}_{ik}^B, c_{i,-k}^B\big)\right)c_{ik}^B.$$

Incentive compatibility requires that $c_{ik}^B \in \arg\max_{\widehat{c}_1^B} \pi_{ik}^B(\widehat{c}_{ik}^B; c_{ik}^B)$, which implies that $\frac{\partial \pi_{ik}^B(c_{ik}^B)}{\partial c_{ik}^B} = -q_{ik}(c_i^A, c_{ik}^B, c_{i,-k}^B)\bar{Q}_i\big(\widehat{c}(c_i^A, c_{ik}^B, c_{i,-k}^B)\big)$, where $\pi_{ik}^B(c_{ik}^B) := \pi_{ik}^B(c_{ik}^B; c_{ik}^B)$. It is clear that the prime contractor will set $t_{ik}(c_i^A, \overline{c}^B, c_{i,-k}^B) = 0$. Therefore, we have that

$$\pi_{ik}^B\left(c_{ik}^B\right) = \int_{c_{ik}^B}^{\overline{c}^B} q_{ik}\left(c_i^A, x, c_{i,-k}^B\right)\bar{Q}_i\left(\widehat{c}\big(c_i^A, x, c_{i,-k}^B\big)\right)dx,$$

which implies that

$$
\begin{aligned}
t_{ik}\left(c_i^A, c_{ik}^B, c_{i,-k}^B\right) = & \, q_{ik}\left(c_i^A, c_{ik}^B, c_{i,-k}^B\right) \bar{Q}_i\left(\widehat{c}\left(c_i^A, c_{ik}^B, c_{i,-k}^B\right)\right) c_{ik}^B \\
& + \int_{c_{ik}^B}^{\bar{c}^B} q_{ik}\left(c_i^A, x, c_{i,-k}^B\right) \bar{Q}_i\left(\widehat{c}\left(c_i^A, x, c_{i,-k}^B\right)\right) dx.
\end{aligned}
\tag{A8}
$$

Since the prime contractor can set the transfer rule according to (A8) for a given bidding rule to ensure incentive compatibility, we consider its optimal choice of the bidding rule. As is standard in the mechanism design literature, the bidding rule also needs to be increasing in c_1^B for the incentive compatibility constraint to be satisfied. We ignore the monotonicity constraint for now and show below that it is satisfied in the optimum. The prime contractor's expected payoff, with the expectation taken over $(c_{i1}^B, \ldots, c_{iN_{Bi}}^B)$, is

$$
\mathbb{E}_{c_{i1}^B,\ldots,c_{iN_{Bi}}^B}\left[\int_{\widehat{c}\left(c_i^A, c_{i1}^B,\ldots,c_{iN_{Bi}}^B\right)}^{\infty}\left(x - c_i^A\right) dQ_i(x) - \sum_{k=1}^{N_{Bi}} t_{ik}\left(c_i^A, c_{ik}^B, c_{i,-k}^B\right)\right].
$$

By plugging (A4) into the expression and changing the order of integration, the prime contractor's expected payoff can be rewritten as

$$
\mathbb{E}_{c_1^B}\left\{\int_{\widehat{c}\left(c_i^A, c_{i1}^B,\ldots,c_{iN_{Bi}}^B\right)}^{\infty}\left[x - c_i^A - \sum_{k=1}^{N_{Bi}} q_{ik}\left(c_i^A, c_{ik}^B, c_{i,-k}^B\right)\tilde{c}_{ik}^B\right] dQ_i(x)\right\}.
$$

Evidently, it is optimal for firm (A, i) to set $q_{ik}\left(c_i^A, c_{ik}^B, c_{i,-k}^B\right) = \mathbb{1}_{\left\{c_{ik}^B < c_{ik'}^B, \forall k' \neq k\right\}}$ and bid $\widehat{c}_i^*\left(c_i^A, c_{i1}^B, \ldots, c_{iN_{Bi}}^B\right) = c_i^A + \min_{1\leq k\leq N_{Bi}} \tilde{c}_{ik}^B$. □

Proof of Proposition 6

Proof. Consider a realization of $\min_{1\leq i\leq N_A}\left(c_i^A + \min_{1\leq k\leq N_{Bi}} \tilde{c}_{ik}^B\right) < \bar{c}^{AB}$ and denote it by x. If $c_{(2)}^{AB} \leq x$, the procurement price in the combinatorial auction is the same as that in holistic procurement. If $c_{(1)}^{AB} \leq x$ and $c_{(2)}^{AB} > x$, the procurement price in the combinatorial auction is $\min_{1\leq i\leq N_A}(c_i^A + \min_{1\leq k\leq N_{Bi}} c_{ik}^B)$—lower than that in holistic procurement,

$\min_{1\le i\le N_A}\left(c_i^A + \min_{1\le k\le N_{Bi}} \tilde{c}_{ik}^B\right) = x$. As a result,

$$\lim_{N_{AB}\to\infty} \frac{\mathbb{E}\left[P^{SPA} - P^{VCG} \middle| \min_{1\le i\le N_A}\left(c_i^A + \min_{1\le k\le N_{Bi}} \tilde{c}_{ik}^B\right) = x\right]}{\Pr\left(c_{(1)}^{AB} \le x, c_{(2)}^{AB} > x\right)}$$
$$= x - \min_{1\le i\le N_A}\left(c_i^A + \min_{1\le k\le N_{Bi}} c_{ik}^B\right) > 0.$$

Under the assumption that $\underline{c}^A + \underline{c}^B \ge \underline{c}^{AB}$, $\Pr(x < c_{(1)}^{AB}) \to 0$ when $N_{AB} \to +\infty$. Therefore, taking expectation over $x = \min_{1\le i\le N_A}\left(c_i^A + \min_{1\le k\le N_{Bi}} \tilde{c}_{ik}^B\right)$ yields the desired result. □

Proof of Proposition 7

Proof. As $N_{Bi} \to \infty$ for all $1 \le i \le N_A$, both $\min_{1\le k\le N_{Bi}} c_{ik}^B$ and $\min_{1\le k\le N_{Bi}} \tilde{c}_{ik}^B$ approach $\underline{c}^B$. The information friction within a consortium effectively disappears. Then Proposition 1 applies and holistic procurement leads to a lower expected procurement price. □